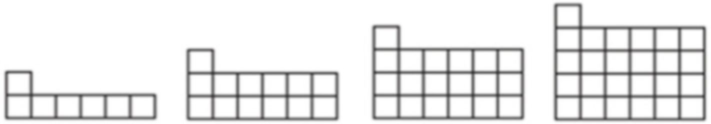
Unit 0: Big Ideas



My notes:

Teacher Notes:

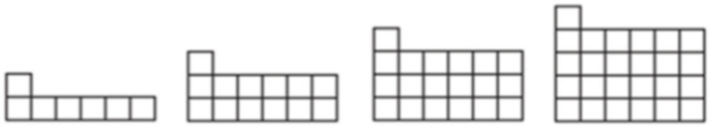
Unit 0: Culture + Habits of Mind

Unit 0: Culture + Habits of Mind

Unit 0: Culture + Habits of Mind

	Skills and Facts
1	I can listen carefully to see how other people reason about problems
2	I can use precise mathematical language
3	I can use precise mathematical notation
4	I can find and identify patterns
5	I know that learning potential is not "fixed." Anyone can learn math if they work at it
6	I can contribute to creating a classroom culture, in which it is safe to take risks and make mistakes
7	I know that strong problem solvers identify and cultivate specific strategies and habits. I can identify and practice some of these strategies.

Unit 0: Big Ideas



My notes:

Teacher notes:

- It is important to listen to how others make sense of their work; listening carefully is how we arrive at common understandings
- Speaking and using precise language and notation is important in mathematics
- Mathematics is a study of patterns and relationships.
- The only way that we can learn is by taking risks, and inevitably making mistakes; our classroom has to support this idea by being a safe place to experiment and make mistakes.
- We all contribute to our classroom culture; we are each valuable and valued parts of a thinking mathematical community
- Mathematicians identify, practice and cultivate specific habits

Unit 0: Culture + Habits of Mind

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Algebra Learning Agreements 2016-17

Responsibilities	Rights	Respect
We are responsible for ourselves and our learning We will bring it! We are responsible to and for each other We are responsible to and for our community We will keep an open mind, and consider all ideas We are responsible to share our ideas	We have the right to learn We have the right to be and to feel safe We have the right to be listened to and to be heard We have the right to have fun We have the right to work at our own pace	We respect ourselves We respect Each other We respect and take care of our space We respect and care for our materials We will demonstrate our respect for others by listening

Our classroom goal is: ____

CCSS Math Practices: The habits of strong mathematical thinkers

Standard	What does it mean?
1. **Make sense of problems and persevere in solving them**	
2. **Reason abstractly and quantitatively**	
3. **Construct viable arguments and critique the reasoning of others**	
4. **Model with mathematics**	

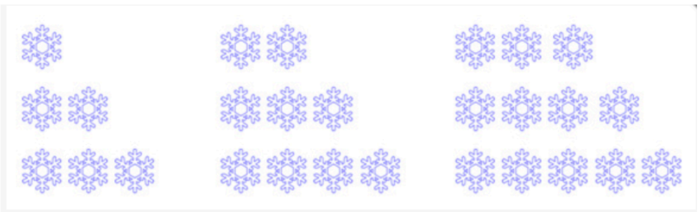
Standard	What does it mean?
5. **Use appropriate tools strategically**	
6. **Attend to precision**	
7. **Look for and make use of structure**	
8. **Look for and express regularity in repeated reasoning**	

CCSS Math Practices: The habits of strong mathematical thinkers

Standard	What does it mean?
1. **Make sense of problems and persevere in solving them**	• Understand the problem, find a way to attack it, and work until it is done. • Allow wait time • Work for progress and "aha" moments. • The math becomes about the process and not about the one right answer.
2. **Reason abstractly and quantitatively**	• *Contextualize* and *decontextualize.* • Break a problem apart and show it symbolically, with pictures, or in any way other than the standard algorithm. • Draw representations of problems
3. **Construct viable arguments and critique the reasoning of others**	• Be able to talk about math, using mathematical language, to support or oppose the work of others. • Post and practice precise mathematical vocabulary
4. **Model with mathematics**	• Use math to solve real problems, organize data, and understand the world around you. • Math limited to math class is worthless. • Use math in science, art, music, and even reading. • Use real graphics, articles, and data from the newspaper or other sources to make math relevant and real

Standard	What does it mean?
5. **Use appropriate tools strategically**	• Select the appropriate math tool to use and use it correctly to solve problems. (eg. Desmos, Calculator, ruler, protractor) • In the real world, no one tells you that it is time to use the meter stick instead of the protractor.
6. **Attend to precision**	• Speak and solve mathematics with exactness and meticulousness.
7. **Look for and make use of structure**	• Find patterns and repeated reasoning that can help solve more complex problems.
8. **Look for and express regularity in repeated reasoning**	• Keep an eye on the big picture while working out the details of the problem. • Generalize – learn to transfer the learning from the problem you're given.

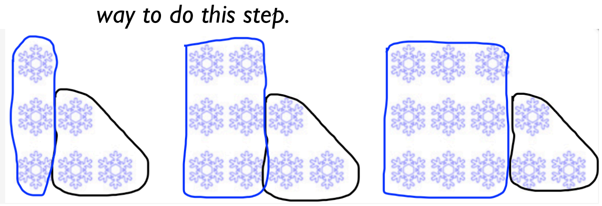
Visual Patterns: Guidelines:



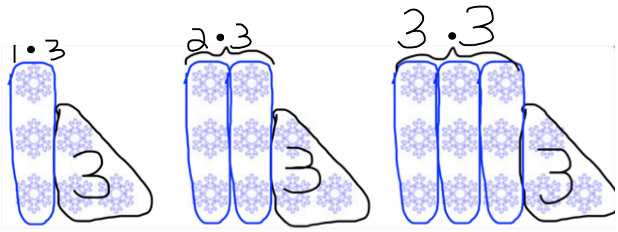
Pattern #9, Snowflakes in step 43 = 132

Step 1: Divide the shape visually into parts.

- *Pro-tip #1: Make rectangles out of the parts that change for easy multiplication!*
- *Pro-tip #2: There is always more than one correct way to do this step.*



Step 2: identify which parts are changing and how.



Step 3: Make a table with what you know

Step	Number of <u>Snowflakes</u>	Parts
1	**6**	$(1 \cdot 3) + 3$
2	**9**	$(2 \cdot 3) + 3$
3	**12**	$(3 \cdot 3) + 3$
4		
5		
6		
10		
43	**132**	
n		

Step 4: Compare the parts to the step number and use your analysis to write an equation and fill in the rest of your table

Step	Number of <u>Snowflakes</u>	Parts
1	**6**	$(1 \cdot 3) + 3$
2	**9**	$(2 \cdot 3) + 3$
3	**12**	$(3 \cdot 3) + 3$
4	15	$(4 \cdot 3) + 3$
5	18	$(5 \cdot 3) + 3$
6	21	$(6 \cdot 3) + 3$
10	24	$(10 \cdot 3) + 3$
43	**132**	$(43 \cdot 3) + 3$
n	$\mathbf{3n + 3}$	$(n \cdot 3) + 3$

Step 5: Check your equation by plugging in 43 to see if it works.

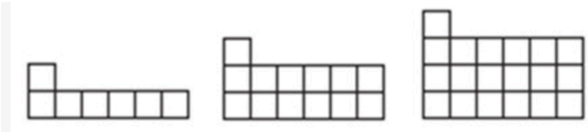
$$\mathbf{3n + 3 \gg 3(43) + 3 \gg 129 + 3 \gg 132}$$

Visual Patterns: Guidelines:

Pattern #14, from Katie, Squares in step 43 = 259

Step 1: Divide the shape visually into parts.

- *Pro-tip #1: Make rectangles out of the parts that change for easy multiplication!*
- *Pro-tip #2: There is always more than one correct way to do this step.*



Step 2: identify which parts are changing and how.

Step 3: Make a table with what you know

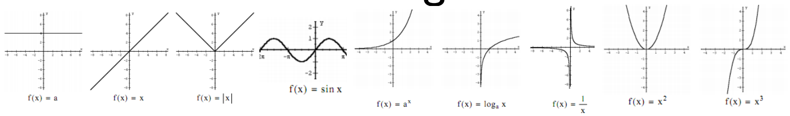
Step	Number of ______	Parts
1		
2		
3		
4		
5		
6		
10		
43		
n		

Step 4: Compare the parts to the step number and use your analysis to write an equation and fill in the rest of your table

Step	Number of ______	Parts
1		
2		
3		
4		
5		
6		
10		
43		
n		

Step 5: Check your equation by plugging in 43 to see if it works.

Unit 1: Big Ideas



My notes:

Teacher notes:

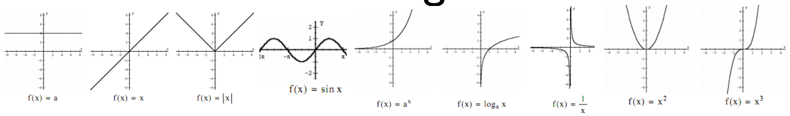
Unit 1: Families of Functions

Unit 1: Families of Functions

Unit 1: Families of Functions

	Skills and Facts
1	I can analyze key features of graphs, including: Increasing interval, decreasing interval, intercepts, periodicity, minimum/maximum, domain and range, end behavior
2	I can recognize the differences between average rate of change vs. Constant Rate of Change
3	I can relate graphs and tables to situations; tell the “story” of a graph
4	I can create scatterplots and Time-Distance Graphs
5	I can distinguish between function families
6	I can ecognize transformations from a parent function
7	I can make connections between multiple representations
8	I understanding that mathematical models can illustrate and reveal aspects of real situations
9	I can Describe key features of functions using appropriate vocabulary
10	I can identify the general shape and behavior of different function families

Unit 1: Big Ideas



My notes:

Teacher notes:

- A function is a correspondence between two sets, X and Y, in which each element of X is matched to one and only one element of Y. The set X is called the domain of the function.
- Function families share similar graphs, behaviors, and properties; functions within a family are transformations of the parent function
- Functions can be represented in multiple, equivalent ways. Each representation has its own advantages
- Mathematical models can illustrate and reveal aspects of real situations; graphing assists in our analysis and understanding.
- The grammar and vocabulary of math, including function notation, allow us to communicate precisely. We can make explicit use of this precision to make strong arguments

Unit 1: Families of Functions

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10	I can identify the general shape and behavior of different function families

Unit 1 Vocabulary	
Relation	A relation is an association between two sets of quantities or information. Example: Any set of ordered pairs. (remember when you are wearing blue **and** white and don't know where to go)
Function	A function is a relation where no two pairs have the same first element.
Domain	All possible *inputs* to a relation or function
Range	All possible *outputs* from a relation or function
Roots (Zeros)	In mathematics, a zero, also sometimes called a root of a function f is a member x of the domain of f such that f(x) = 0. In other words, a "zero" of a function is an input value that produces an output of zero (0)
Discrete	Discrete data refers to data that can only take certain values. Example: the number of students in a class (you can't have half a student!).
Continuous	A set of data is said to be continuous if the values belonging to the set can take on ANY value within a finite or infinite interval
Turning Point	A *turning point* of a polynomial is a point where a function changes from an increasing interval to a decreasing interval or vice versa.
Interval	In mathematics, a (real) interval is a set of real numbers with the property that any number that lies between two numbers in the set is also included in the set
Increasing/Decreasing Intervals	Intervals of increase and decrease are the domain of a function where its value is getting larger or smaller, respectively
End Behavior	The end behavior of a polynomial function is the behavior of the graph of $f(x)$ as x approaches positive infinity or negative infinity.
Symmetry	Symmetry is when one shape becomes exactly like another if you flip, slide or turn it.
Intercept	The point or coordinates at which a line, curve, or surface intersects a coordinate axis.

*Don't forget to use your Key Feature Cards for more info and examples!

Function Families

For Unit 1, you will be asked to identify key features from each of these function families

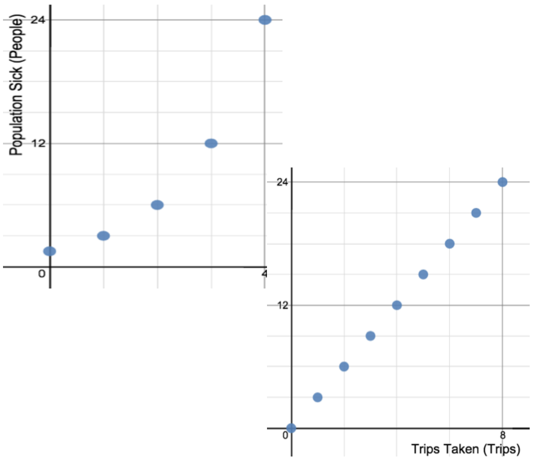
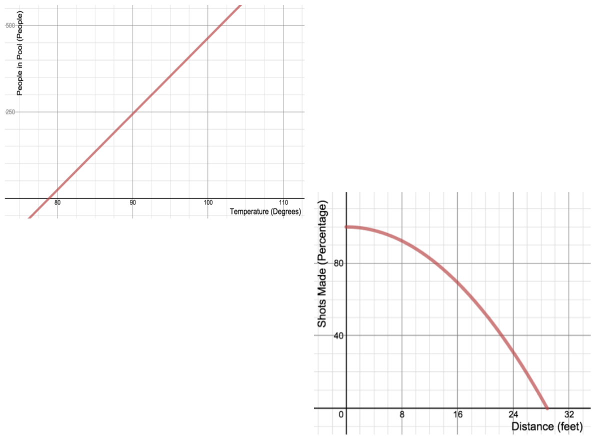
- Linear Functions
- Exponential Functions
- Absolute Value Function
- Periodic Functions
- Quadratic Functions
- Polynomial Functions
- Square Root Functions
- Rational Functions
- Logarithmic functions
- Piecewise Functions
- Cubic Functions

Characteristics of Function Families

- The graphs in each function family share similar characteristics and shapes
- The equations in a function family have a similar form - the **parent** of the family is the equation in the family with the simplest form.
    - Example, $f(x) = x$ is a parent to other functions, such as $f(x) = 3x - 9$
    - Example, $f(x) = x^2$ is a parent to other functions, such as $f(x) = 2x^2 - 5x + 3$.
- The way that the rate of change behaves within a function family follows similar patterns

KEY FEATURE CARDS

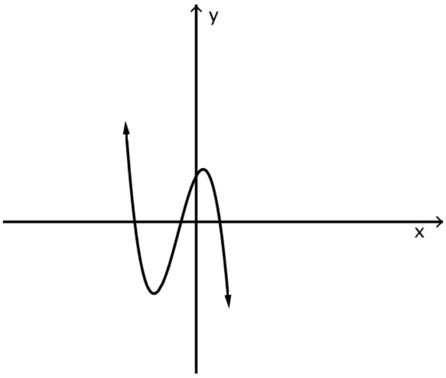
Directions: Analyze the Key Feature card. Use what you notice to define each term and use examples from the graphs to support your definition.

<table>
<tr><th colspan="2">Discrete Vs. Continuous</th></tr>
<tr>
<td>GRAPH 1
These graphs represent Discrete situations

</td>
<td>GRAPH 2
These graphs represent Continuous situations

</td>
</tr>
<tr><td colspan="2">DEFINITION:</td></tr>
<tr><td colspan="2">How can you tell if a situation is discrete or continuous?</td></tr>
</table>

KEY FEATURE CARD: Zeros

ZEROS or ROOTS

GRAPH 1

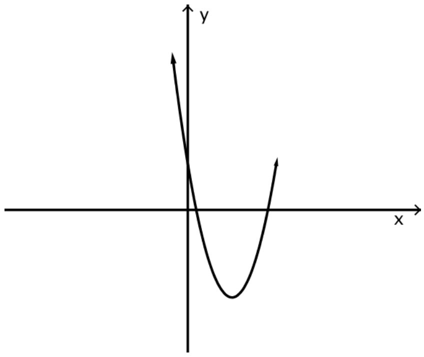


This graph has three distinct real zeros

$as \quad x \to -\infty, \ f(x) \to \infty$

$as \quad x \to \infty, \ f(x) \to -\infty$

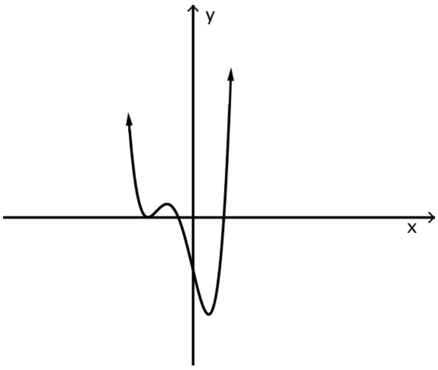
GRAPH 2



This graph has two distinct real zeros

$as \quad x \to -\infty, \ f(x) \to \infty$

$as \quad x \to \infty, \ f(x) \to \infty$



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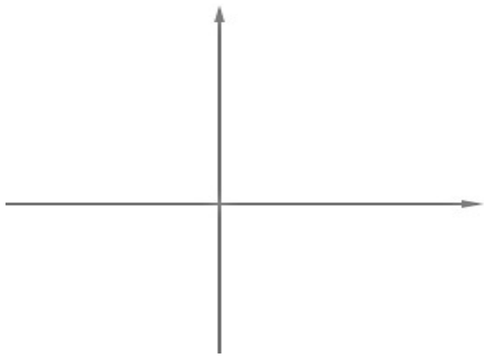


This graph has four distinct real zeros

$as \quad x \to -\infty, \ f(x) \to -\infty$

$as \quad x \to \infty, \ f(x) \to -\infty$

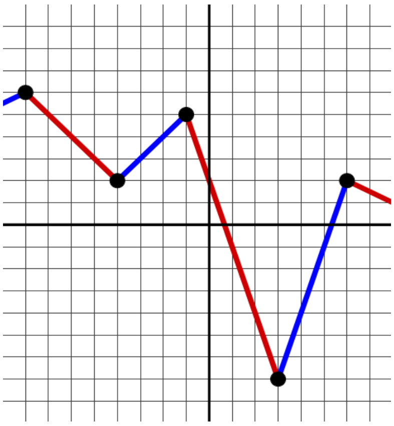
DEFINITION:

KEY FEATURE CARDS

Directions: Analyze each of the Key Feature cards. Use what you notice to define each term and use examples from the graphs to support your definition.

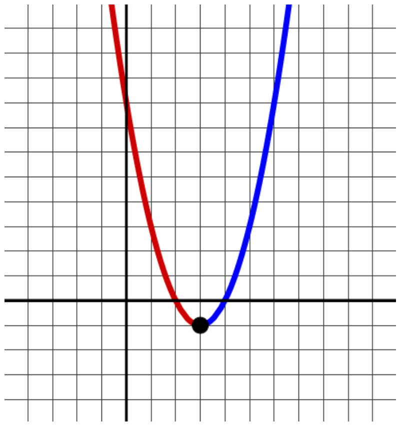
TURNING POINT

GRAPH 1



This graph has five turning points.

GRAPH 2

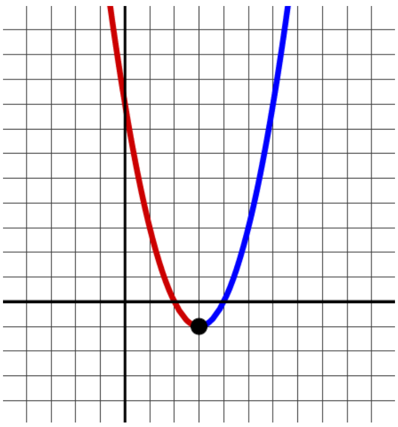


This graph has one turning point.

DEFINITION:

INCREASING and DECREASING INTERVALS

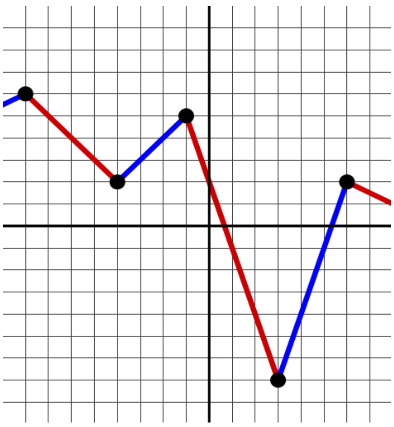
GRAPH 1



This graph is ***decreasing*** when x is less than 3

This graph is ***increasing*** when x is greater than 3.

GRAPH 2



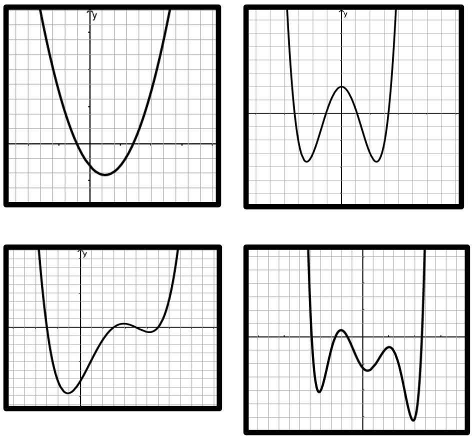
This graph is ***decreasing*** when x is between -8 and -4 or -1 and 3 or more than 6.

This graph is ***increasing*** when x is less than -8, from -4 to -1 and from 3 to 6.

DEFINITION:

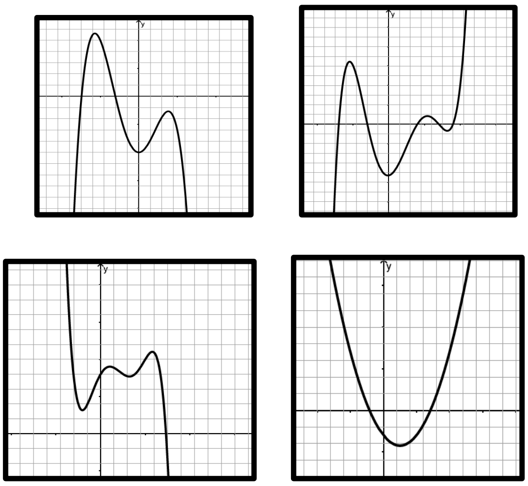
END BEHAVIOR

GRAPHS SET A



All of the graphs have **<u>the same</u>** end behavior.

GRAPHS SET B

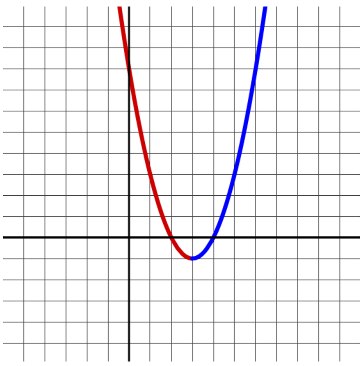


All of the graphs have **<u>different</u>** end behavior.

DEFINITION:

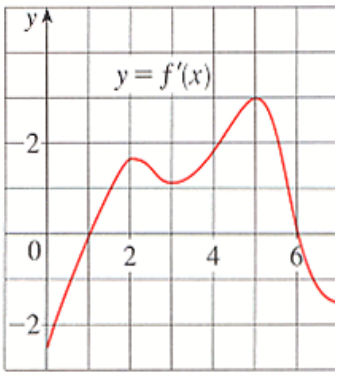
SYMMETRY

GRAPH 1



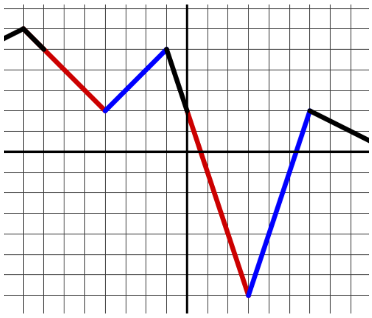
This graph is symmetrical over the line x = 3

GRAPH 2



This graph does not have any lines of symmetry.

GRAPH 3

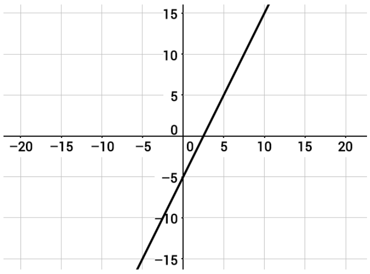
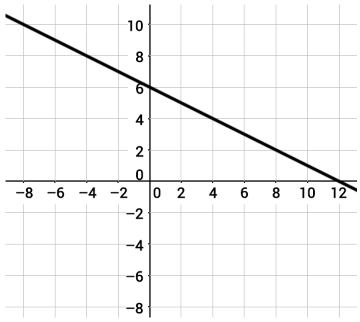


When $-7 \le x \le -1$, the graph is symmetrical over the line x = -4. Also, when $0 \le x \le 6$, the graph is symmetrical over the line x = 3.

DEFINITION:

FUNCTION FAMILY CARDS

Linear Functions: Parent $f(x) = x$

Example 1	Example 2	Example 3
(table below)		

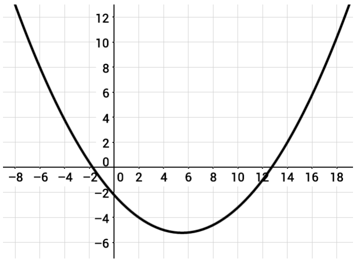
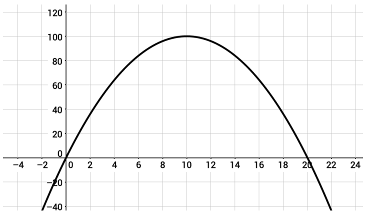
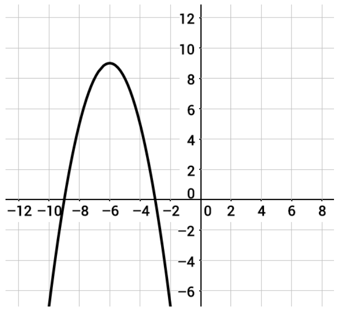
x	y
1	2
2	5
3	8
4	11

How does the rate of change behave in Linear Functions?

Significant features:

Examples

Quadratic Functions: Parent $f(x) = x^2$

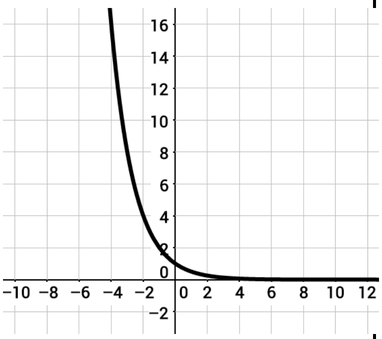
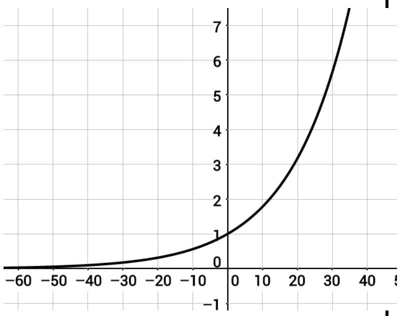
Example 1	Example 2	Example 3
		

How does the rate of change behave in Quadratic Functions?

Significant features:

Examples

Exponential Functions: Parent $f(x) = b^x$

Example 1	Example 2	Example 3
		see table below

Example 3

x	y
0	1
1	3
2	9
3	27
4	81

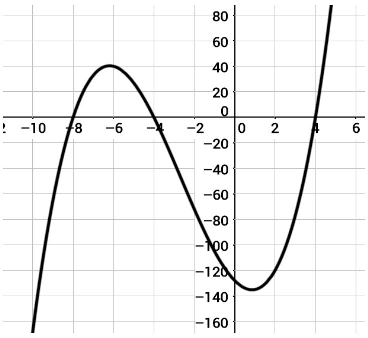
How does the rate of change behave in Exponential Functions?

Significant features:

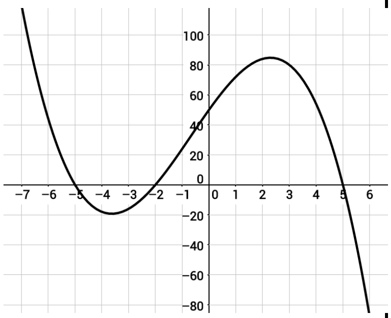
Examples

Cubic Functions: Parent $f(x) = x^3$

Example 1



Example 2



Example 3

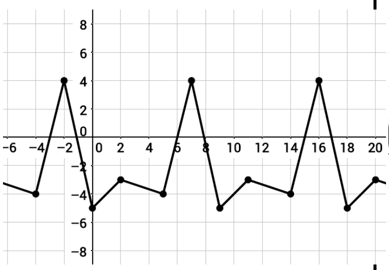
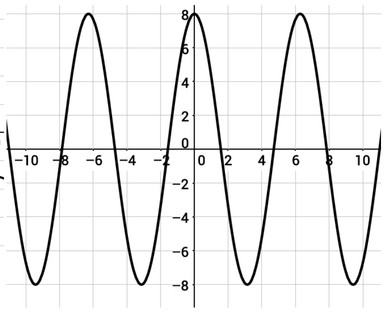
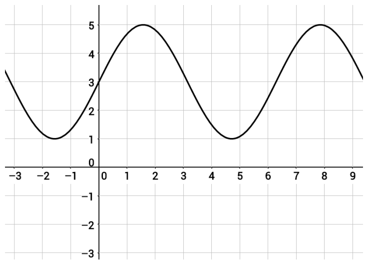
x	y
-1	-1
0	0
1	1
2	8
3	27

How does the rate of change behave in Cubic Functions?

Significant features:

Examples

Periodic Functions: (Parent varies)

Example 1	**Example 2**	**Example 3**
		

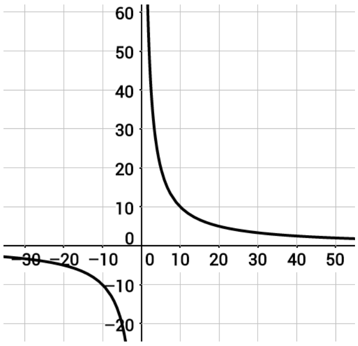
How does the rate of change behave in Periodic Functions?

Significant features:

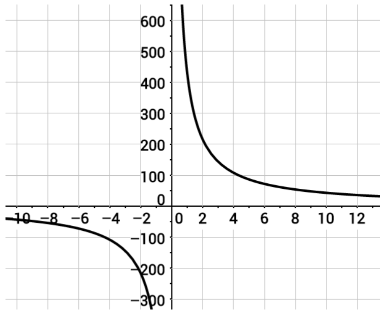
Examples

Rational Functions: Parent $f(x) = \frac{1}{x}$

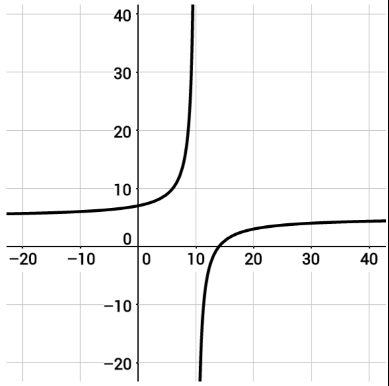
Example 1



Example 2



Example 3

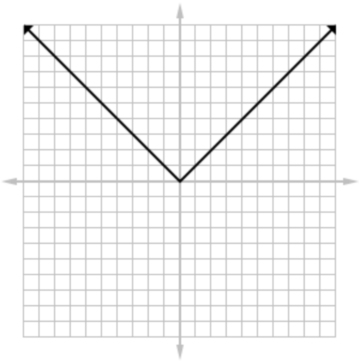
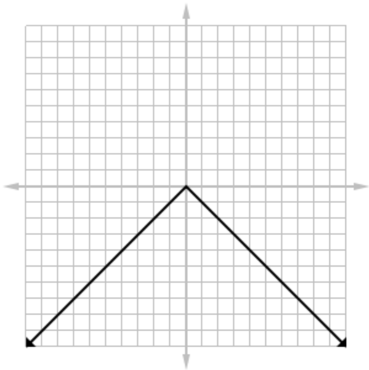
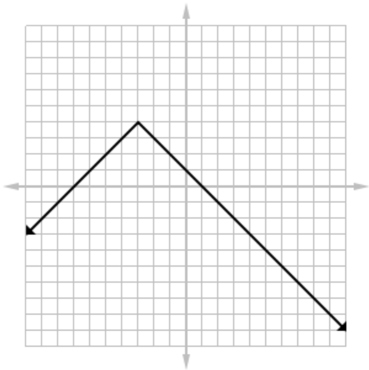


How does the rate of change behave in Rational Functions?

Significant features:

Examples

Absolute Value Functions: Parent $f(x) = |x|$

Example 1	**Example 2**	**Example 3**
		

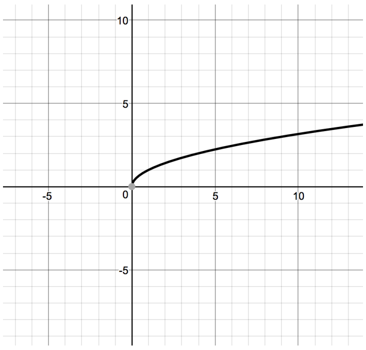
How does the rate of change behave in Absolute Value Functions?

Significant features:

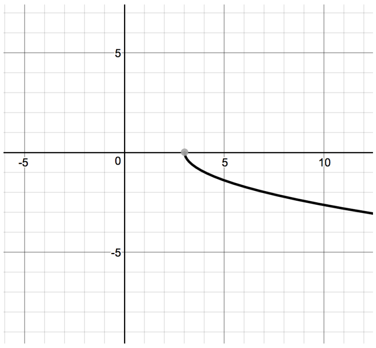
Examples

Square Root Functions: Parent $f(x) = \sqrt{x}$

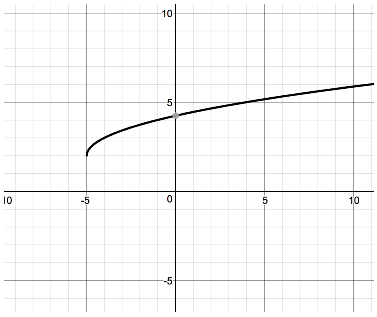
Example 1



Example 2



Example 3

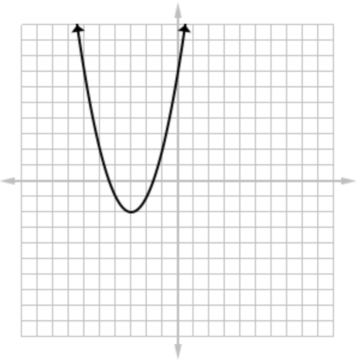
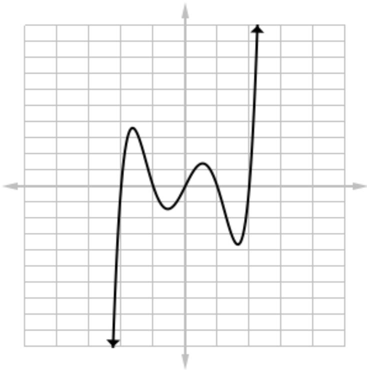
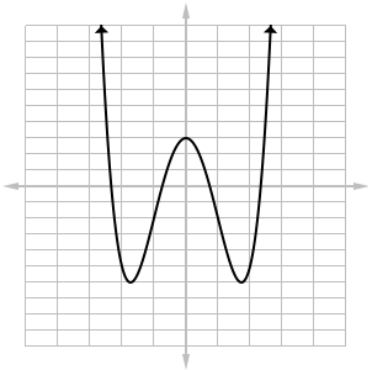


How does the rate of change behave in Square Root Functions?

Significant features:

Examples

Polynomial Functions: (Parent varies)

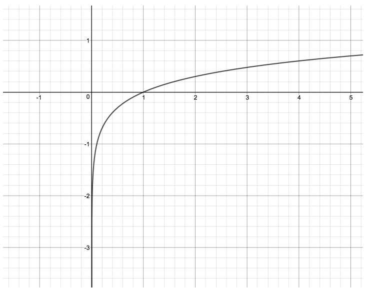
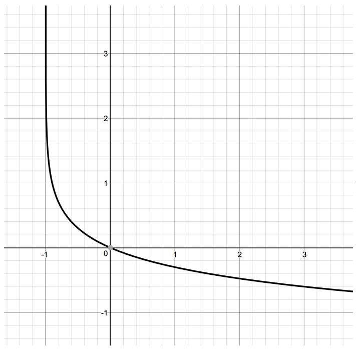
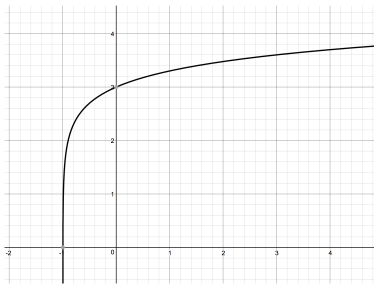
Example 1	Example 2	Example 3
		

How does the rate of change behave in Polynomial Functions?

Significant features:

Notes: Quadratics and Cubic functions are just specific and special types of polynomial functions.

Logarithmic Functions: Parent $f(x) = log(x)$

Example 1	Example 2	Example 3
		

How does the rate of change behave in Logarithmic Functions?

Significant features:

Examples

Situations to Match to Function Family Cards

Taxi cabs fare: cost vs. miles
The volume of water filling a tank over time
The height of a thrown ball as it changes over time
The number of candies left in a jar if the same amount is eaten each day
The population of bacteria, which doubles every four hours as it grows over time
Any situation that increases at a constant rate
The phases of the moon over time
A situation with only one maximum or high point or one low or minimum point
The amount of daylight per day as it changes over the year
Amount of money in a bank savings account at a fixed interest rate
A situation where the output is being multiplied repeatedly as the input increases
A situation where the output is being divided repeatedly as the input increases
The amount of air in a person’s lungs over time while they breathe regularly
The height of a bicyclist’s right foot over time (While she rides the bike!)
Volume of a sphere as the radius changes
Any situation that decreases at a constant rate

The change in the ocean tide at a beach over time
The length of the hypotenuse of a right triangle when you know the sum of the squares of the other two sides
How far off your guess is when you guess how many jelly beans there are in a jar
Number of candies each kid will get by dividing a big bag of candies depending on how many kids there are
Phone service: cost vs. minutes used
Cost per person of a chartered bus if the bus has a fixed price, which is divided evenly amongst all of the riders
You want to build a square swimming pool that has an area of x square feet. How long should the sides be?
Truck driving at a constant speed: distance vs. time
Roller coaster designers need a function to describe the multiple curves in their rides
Combinations of polynomial functions are used in economics to do cost analyses

Applications or Real Life Situations
After students the situation to the family they can optionally rewrite a more detailed description with numbers. The goal is to group to the family not the individual function graph. After having time to work for a while students should be asked to think about discrete vs. continuous in a brief discussion and then back to work in groups.

1. **Linear Functions**
   a. taxi cabs fares, cost vs. time (discrete)
   b. phone service, bill vs. minutes used
   c. a truck driving a constant speed, distance vs. time
   d. number of candies left in a jar if the same amount is eaten each day
   e. any situation that increase or decreases at a constant rate

2. **Quadratic Functions**
   a. if the length and width of a increases the same amount the area changes
   b. the height of a thrown ball as it changes over time
   c. a situation with only one maximum or high point or one low or minimum point

3. **Exponential Functions**
   a. population of bacteria as it grows over time
   b. growth of an embryo of an organism in the first few hours
   c. a bank savings account at a fixed interest rate
   d. number of people who read a popular Tweet on Twitter and retweet it over a couple of days
   e. any situation where the output is being multiplied or divided as the input increases

4. **Periodic Functions**
   a. amount of air in a person's lungs over time while they breathe regularly
   b. the height of a bicyclist's right foot over time
   c. the amount of daylight as it changes over the year
   d. the phases of the moon over time
   e. the change in the ocean tide at a beach over time

5. **Cubic Functions**
   a. the volume of water filling a tank over time
   b. volume of a sphere as the radius changes

6. **Polynomial Functions**
   a. Combinations of polynomial functions are used in economics to do cost analyses
   b. Roller coaster designers use polynomials to describe the curves in their rides
7. **Rational Functions**
   a. Cost per person of a chartered bus if the bus has a fixed price, which is divided evenly amongst all of the riders
   b. number of candies each kid will by dividing a big bag of candies depending on how many kids there are

8. **Absolute (linear) Functions**

a. How far off your guess is when you guess how many jelly beans there are in a jar

9. **Square Root Functions**

   a. The length of the hypotenuse of a right triangle when you know the sum of the squares of the other two sides

   b. You want to build a square swimming pool that has an area of x square feet. How long should the sides be?

[Brackets] vs. (Parenthesis)

We use brackets and parenthesis to indicate that we are talking about a specific section of the Domain (x-values) of a function.

Use a bracket to indicate that the endpoint is included in the interval, a parenthesis to indicate that it is not.

- Brackets are like inequalities that say "or equal to" ($\geq or \leq$)
- Parentheses are like strict inequalities. ($> or <$)

Examples:

- The interval (3,7) includes 3.1 and 3.007 and 3.00000000002, but it does not include 3. It also includes numbers greater than 3, but it does not include 7.
- The interval [4,9] includes 4 and every number from 4 up to 9, and it also includes 9
- Mixed intervals (a,b] or [a,b) are also possible.
- The symbols $-\infty$ (and ∞) are used to indicate that there is no left (…or right) endpoint for the interval. They are not endpoints, but indicators that there is no endpoint. They always use parentheses.

Function Notation

Traditionally, functions are referred to by the letter name *f*, but *f* need not be the only letter used in function names. The following are a few of the notations that may be used to name a function:

$$f(x),\ g(x),\ h(a),\ A(t),\ \ldots$$

Note: $f(x)$ notation can be thought of as another way of representing the *y*-value in a function, especially when graphing. The *y*-axis is even labeled as the $f(x)$ axis, when graphing.

Inverse Functions

The inverse of $f(x)$ is written as:

Things to know about inverse functions:

Given the t-chart for $f(x)$, find the t-chart for $f^{-1}(x)$

$f(x)$	
x	**y**
-5	0
-3	-2
1	5
2	-1
3	3
5	6

$f^{-1}(x)$	
x	**y**

Determine if f(x) and g(x) are inverse functions.

1) $f(x) = x^2 + 2 \quad g(x) = \sqrt{x+2}$

2) $f(x) = 3x + 1 \quad g(x) = \dfrac{x-1}{3}$

Find the inverse

3) $f(x) = 2x - 10$

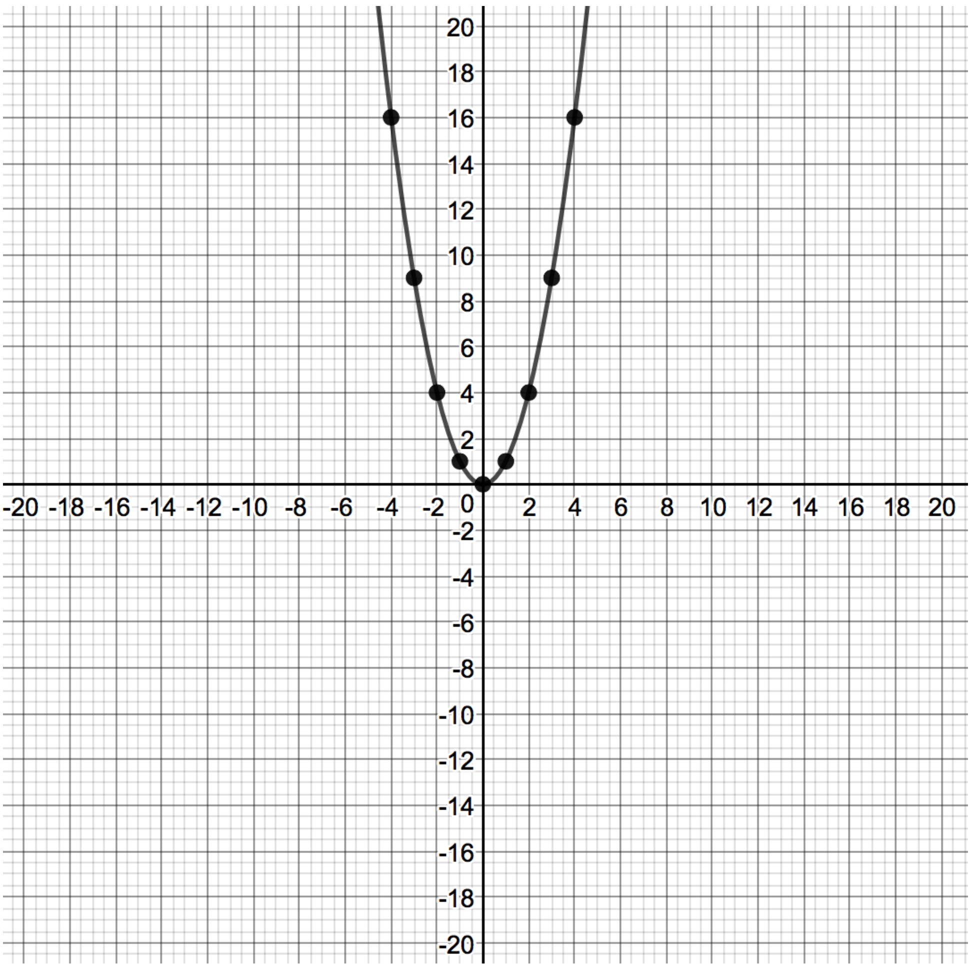
4) $f(x) = x^2 + 3$

Graphs of Inverse Functions

Sketch the graphs, and then answer the following question:
What is true about the graphs of inverse functions?

Given the graph for $f(x)$, sketch the graph for $f^{-1}(x)$ (on the same axes)

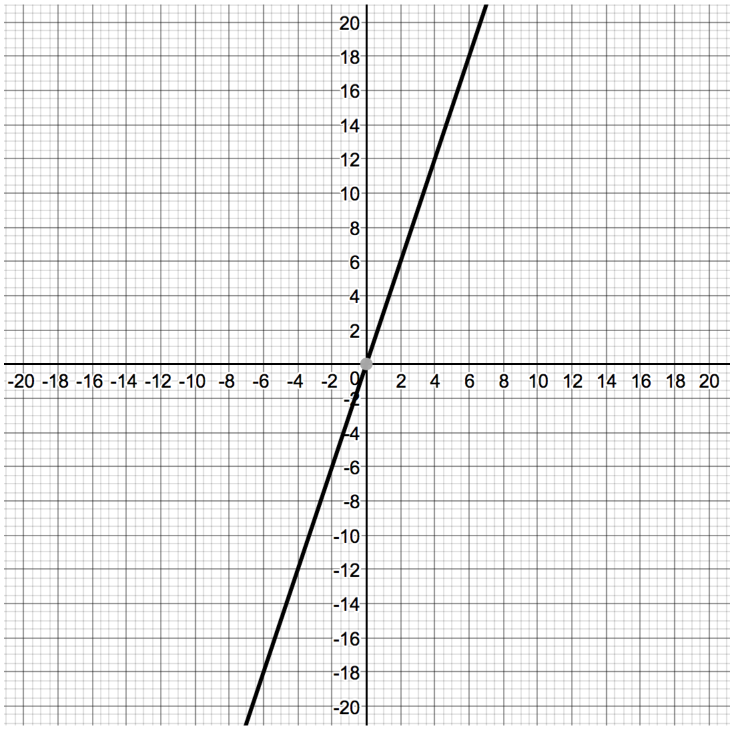
$f(x)=x^2$ *$f^{-1}(x)=$*



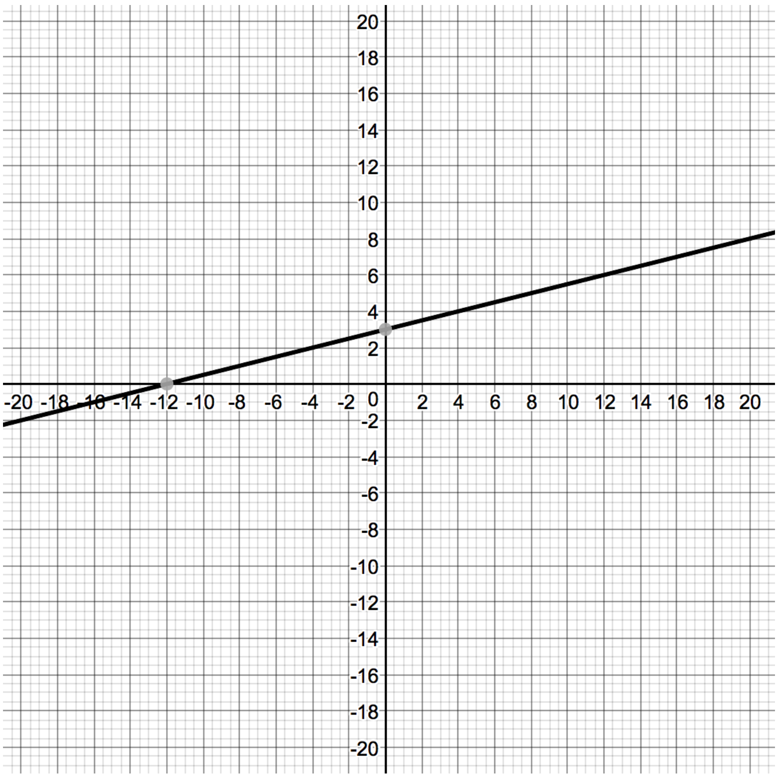
Graphs of Inverse Functions

Given the graph for $f(x)$, sketch the graph for $f^{-1}(x)$ (on the same axes)

f(x)=3x *f⁻¹(x)=*



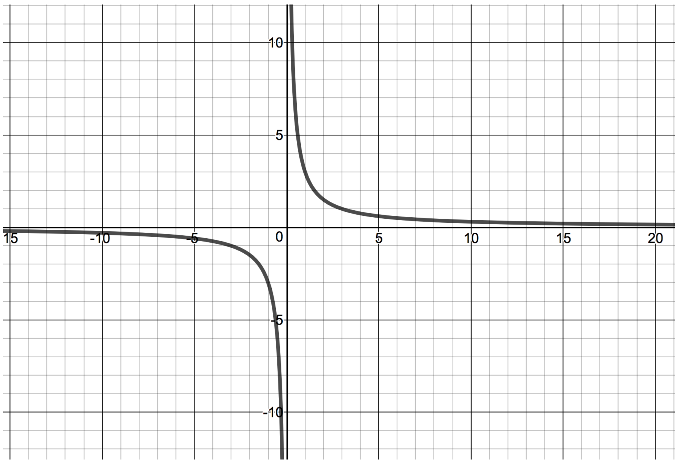
f(x)=1/4 x +3 *f⁻¹(x)=*



INVERSE VARIATION

- Quantities **vary inversely** if they are related by the relationship $y = \frac{k}{x}$
- Another way to express this is $xy = k$
- We also say that **y varies inversely with x**.
- When quantities vary inversely, the constant k is called the **constant of proportionality**.
- Quantities, which vary inversely, are also said to be **inversely proportional**.

EXAMPLE: $y = \frac{3}{x}$



Features of the graph to notice:

- Characteristic shape – the graph is in 2 "parts" (although given a real context, we often only use the part of the function where x>0
- the rate of change gets closer and closer to undefined as x approaches zero, and gets closer and closer to zero as x gets bigger
- The graph approaches the x-axis as x gets large (**end behavior**).

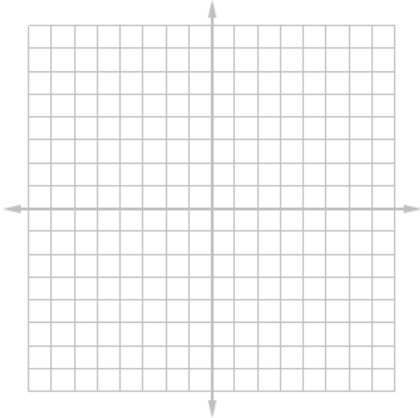
Parent Functions

<table>
<tr><th>Function</th><th>Table of Values</th><th>D/R/Int</th><th>Graph</th></tr>
<tr><td>Linear

$f(x) = x$</td><td><table><tr><th>x</th><th>y</th></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table></td><td>Domain:

Range:

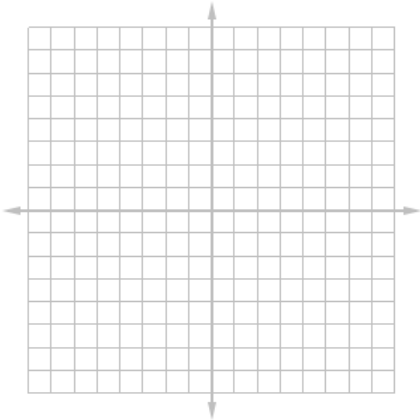
y-int:

x-int:</td><td></td></tr>
<tr><td>Quadratic

$f(x) = x^2$</td><td><table><tr><th>x</th><th>y</th></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table></td><td>Domain:

Range:

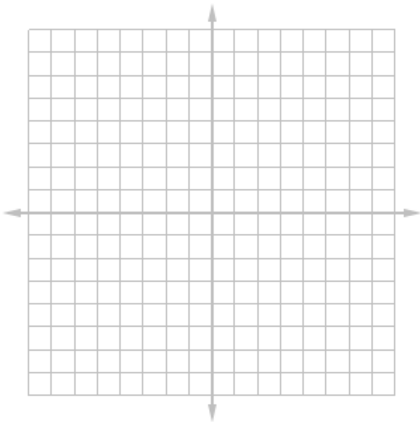
y-int:

x-int:</td><td></td></tr>
<tr><td>Cubic

$f(x) = x^3$</td><td><table><tr><th>x</th><th>y</th></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table></td><td>Domain:

Range:

y-int:

x-int:</td><td></td></tr>
<tr><td>Absolute Value

$f(x) = |x|$</td><td><table><tr><th>x</th><th>y</th></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table></td><td>Domain:

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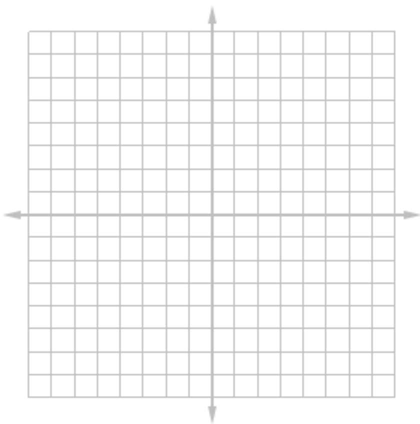
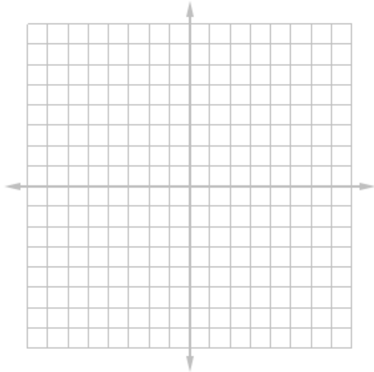
x-int:</td><td></td></tr>
</table>

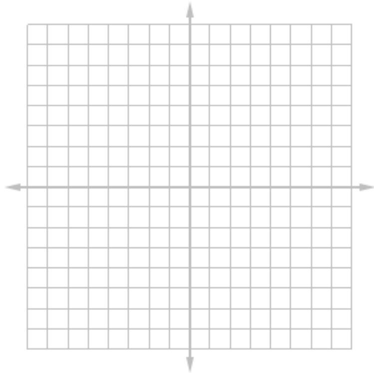
<table>
<tr><th>Function</th><th>Table of Values</th><th>D/R/Int</th><th>Graph</th></tr>
<tr><td rowspan="2">Rational

$f(x) = \frac{1}{x}$</td><td rowspan="2"><table><tr><th>x</th><th>y</th></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table></td><td>Domain:

Range:</td><td rowspan="2"></td></tr>
<tr><td>y-int:

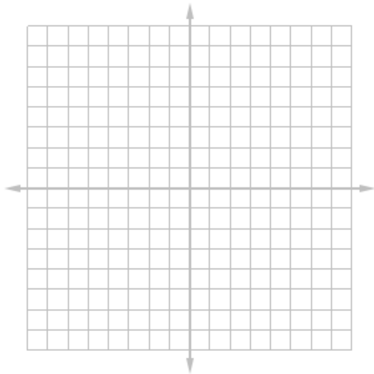
x-int:</td></tr>
<tr><td rowspan="2">Square Root

$f(x) = \sqrt{x}$</td><td rowspan="2"><table><tr><th>x</th><th>y</th></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table></td><td>Domain:

Range:</td><td rowspan="2"></td></tr>
<tr><td>y-int:

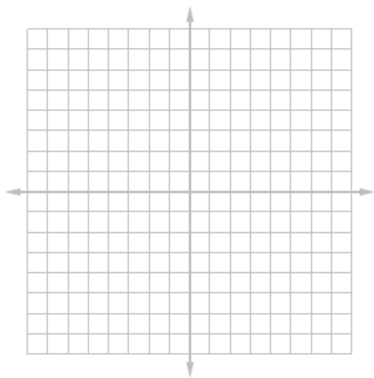
x-int:</td></tr>
<tr><td rowspan="2">Exponential Growth
(a > 1)

$f(x) = a^x$
(Let a=2 for this example)</td><td rowspan="2"><table><tr><th>x</th><th>y</th></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table></td><td>Domain:

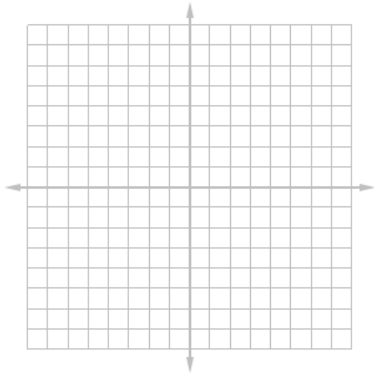
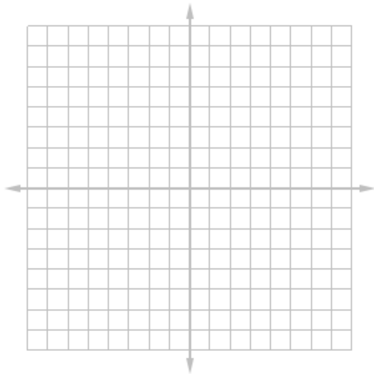
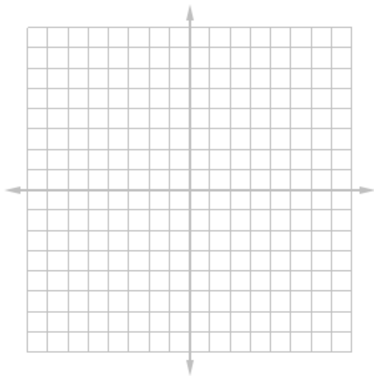
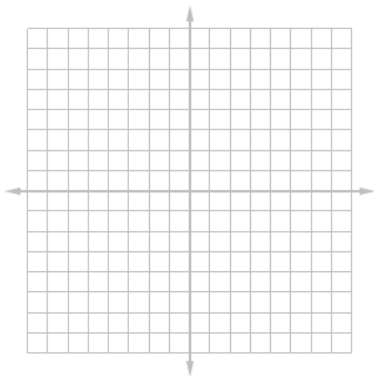
Range:</td><td rowspan="2"></td></tr>
<tr><td>y-int:

x-int:</td></tr>
<tr><td rowspan="2">Exponential Decay
(0 < a < 1)

$f(x) = a^x$
(Let $a = \frac{1}{2}$ for this example)</td><td rowspan="2"><table><tr><th>x</th><th>y</th></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table></td><td>Domain:

Range:</td><td rowspan="2"></td></tr>
<tr><td>y-int:

x-int:</td></tr>
</table>

Function	Table of Values	D/R/Int	Graph
Logarithmic $f(x) = \log_a x$ (let a=10 for this example)	We'll get to the details of these functions later!		
Trigonometric $f(x) = sinx$ (This is just one example)	(We'll get to the details of these functions later! For now, just remember that they are periodic …and awesome)		
Piecewise $f(x) = \begin{cases} x+1, x \leq 0 \\ 3x+1, x > 0 \end{cases}$ (This is just one example)	We'll get to the details of these functions later!		
Polynomial $f(x) = ax^n + bx^{n-1} \ldots + cx^3 + dx^2 + e$ (Again, this is just one example)	We'll get to the details of these functions later!		

TEACHER NOTES

- A function is a correspondence between two sets, *X* and *Y*, in which each element of *X* is matched to one and only one element of *Y*. The set *X* is called the domain of the function.
- Function families share similar graphs, behaviors, and properties; functions within a family are transformations of the parent function
- Functions can be represented in multiple, equivalent ways. Each representation has its own advantages
- Mathematical models can illustrate and reveal aspects of real situations; graphing assists in our analysis and understanding.
- The grammar and vocabulary of math, including function notation, allow us to communicate precisely. We can make explicit use of this precision to make strong arguments

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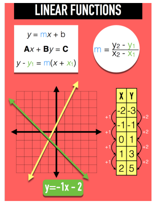


Unit 2: Big Ideas

My notes:

Teacher notes:

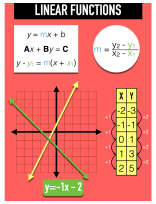
- Linear equations represent real and abstract situations characterized by a constant rate of change. Their graphs always make straight lines
- Equations can be manipulated from one form to another, and different but equivalent forms reveal different aspects of a function
- Function rules describe the quantitative relationships between variables. Tables are useful for beginning a pattern, and nth term expressions are useful for making more distant predictions
- The initial value and rate of change of a linear function can be used to make predictions.
- An absolute value graph has a characteristic “V” shape, and transformations from the absolute value parent function y=|x| share this characteristic. Absolute value represents distance from zero.
- Systems of equations represent situations with more than one constraint, and contain functions that share the same set of variables. A solution simultaneously makes each function rule in a system of equations true, and the solution to a system of equations can be represented in multiple, equivalent ways.
- Linear inequalities in 2 variables divide the plane into two regions. They are related to linear equations, but can be used to model situations with a maximum or minimum

Unit 2: Linear Functions

Unit 2: Linear Functions

Unit 2: Linear Functions

	Skills and Facts
1	I understand that linear functions are characterized by a constant rate of change
2	I can recognize and solve linear combinations
3	I can write and solve linear equations for a given situation
4	I can fit lines to data on a graph (find a line of best fit), through sketching and through technology.
5	I can recognize and solve linear absolute value problems
6	I can describe key features of linear and absolute value functions using appropriate vocabulary
7	I can write explicit and recursive expressions for arithmetic sequences using proper notation
8	I can determine the slope and intercepts of a line given its equation.
9	I can find an equation of a line given two points on it or given a point on it and its slope
10	I can solve mixture problems with Algebra
11	I can graph linear inequalities
12	I can describe linear functions through graphic, algebraic, verbal, and tabular representations
13	I can solve systems of inear equations in two variables



Unit 2: Big Ideas

My notes:

Teacher notes:

Unit 2: Linear Functions

Unit 2: Linear Functions

Unit 2: Linear Functions

	Skills and Facts
1	I understand that linear functions are characterized by a constant rate of change
2	I can recognize and solve linear combinations
3	I can write and solve linear equations for a given situation
4	I can fit lines to data on a graph (find a line of best fit), through sketching and through technology.
5	I can recognize and solve linear absolute value problems
6	I can describe key features of linear and absolute value functions using appropriate vocabulary
7	I can write explicit and recursive expressions for arithmetic sequences using proper notation
8	I can determine the slope and intercepts of a line given its equation.
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Unit 2: Honors-Level Extensions

	Skills and Facts
1	I can use linear programming to solve optimization problems
2	I can solve linear systems with 3 variables
3	I know that geometrically, the absolute value of a number is its distance on a number line from 0; Algebraically, the absolute value of a number equals the nonnegative square root of its square.
4	I can use Sigma notation to describe an arithmetic series with constant or algebraic differences, and I can find the sum of that arithmetic series
5	I can find unknown terms of an arithmetic sequence with constant or algebraic differences

Additional Notes

Unit 2: Linear Functions: Honors-Level Extensions

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Additional Notes

Unit 2: Linear Functions: Honors-Level Extensions

Unit 2: Linear Functions

Essential Questions

What types of relationships can be modeled by linear functions, and what do math models of these relationships look like?
Why are there different forms for notating equations of lines, and how can we decide in which format to write a linear equation?
How can we write recursive or explicit rules for linear situations, and why do we write function rules?
How can we use linear equations to make predictions?
What are the key features of absolute value relationships and graphs?
What are the key features of absolute value relationships and graphs?
What can we do with a system of equations/inequalities that we cannot do with a single equation/inequality?
How do we find solutions to systems?
How are linear inequalities similar or different from linear equations, and when are they useful?

Unit 2: Linear Functions

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How do we find solutions to systems?

How are linear inequalities similar or different from linear equations, and when are they useful?

3 FORMS OF LINEAR EQUATIONS	
Slope-intercept form	
Example:	
Slope:	
y-intercept:	
Other notes:	
Point-slope form	
Example:	
Slope:	
y-intercept:	
Other notes:	
Standard form	
Example:	
Slope:	
y-intercept:	
Other notes:	

ABSOLUTE VALUE

General Form of an **Absolute Value Equation**:

How do the constants change the graph of the parent function $f(x) = |x|$

a: ____

h: ____

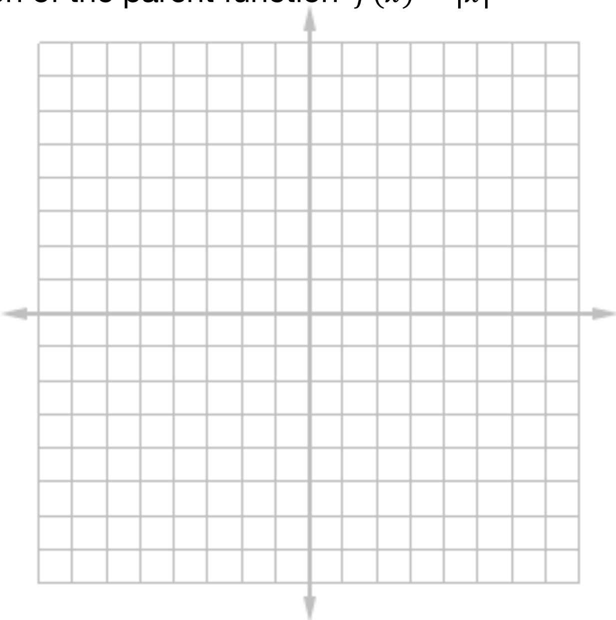
k: ____

If you have an absolute value equation in general form, what are the coordinates of the **turning point** (the "vertex") of the graph?

Example: for the equation $f(x) = \frac{1}{2}|x - 3| + 8$

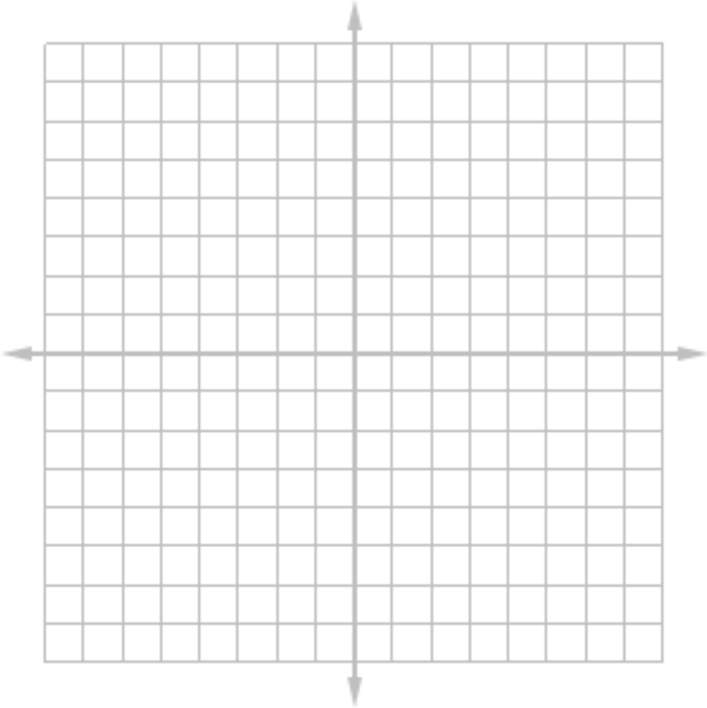
The location of the turning point is $(\quad , \quad)$

Sketch a graph of the parent function $f(x) = |x|$

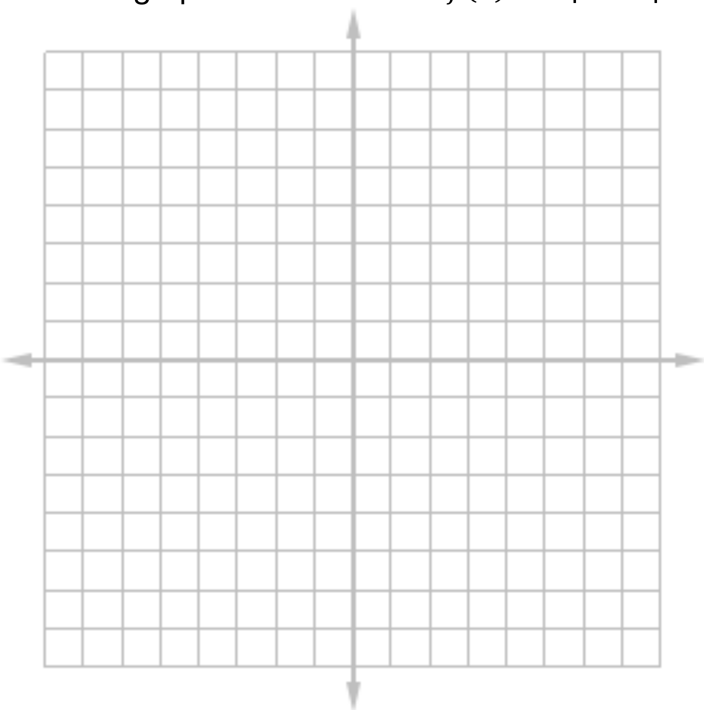


ABSOLUTE VALUE

Sketch a graph of the function $f(x) = |x| - 3$



Sketch a graph of the function $f(x) = -|x - 3| + 1$



ABSOLUTE VALUE

Solving Absolute Value Equations: Try to always think of absolute

value problems in terms of ___________________________

Example: $|x - 25| = 15$

This means that _________ is ________units away from ________

Solve this geometrically:

Solve this with algebra: (remember – set the absolute value equal to the positive solution and the negative solution to find all possibilities)

Example #2: $|5x + 1| = |3x - 1|$

Solve this with algebra:

ABSOLUTE VALUE INEQUALITIES

To solve these problems, combine what you know about solving absolute value equations and linear inequalities. Remember – think distance!

Example 1: $|x - 5| \geq 12$

This is telling us that x is ***at least*** ______ units away from _______.

Treat this like an equality and solve: $|x - 5| = 12$

x= or x=

Option 1: Use algebra to test one or two points:

Option 2: Think geometrically:

Example 2: $|x - 7| < 25$

Unit 2 Vocabulary	
Linear Equation	An **equation** between two **variables** that gives a **straight line** when plotted on a **graph**.
Sequence	A **sequence** is an ordered list of numbers; the numbers in this ordered list are called "**elements**" or "**terms**".
Series	A **series** is the value you get when you add up all the **terms** of a **sequence**; this value is called the **sum**.
Arithmetic Sequence	A **sequence** made by adding the same value each time. Example: 1, 4, 7, 10, 13, 16, 19, 22, ... (each number is 3 larger than the number before it).
Arithmetic Series	The sum of an **arithmetic sequence**.
Common Difference	The difference between any two **consecutive terms** in an **arithmetic sequence**
n^{th} term	By "the **nth term**" of a sequence we mean an expression that will allow us to calculate the **term** that is in the **nth** position of the **sequence**.
System of Equations	A **system of equations** is a collection of two or more **equations** with a same set of unknowns. In solving a **system of equations**, we try to find values for each of the unknowns that will satisfy **every equation** in the **system**.
Inequality	An **inequality** is a **mathematical sentence** that uses symbols such as $<$, $\leq$, $>$, or $\geq$ to compare two quantities.
Absolute Value	The **magnitude** of a **real number** without regard to its sign; How far a number is from zero.

MIXTURE PROBLEMS
Mixture problems are just examples of **Linear Combinations** linear equations written in standard form: $Ax + By = C$
Example: A chemist wants to mix some 70% saline solution with 8 liters of a 25% saline solution to create a solution that is 40% salt. How many liters of the 70% solution does she need?

MIXTURE PROBLEMS

Mixture problems are just examples of **Linear Combinations**
linear equations written in standard form: $Ax + By = C$

Example: Leonard has a 70% saline (salt) solution. He also has some of a 25% saline solution. Leonard wants to take 8 liters of his 70% solution, mix it with his 25% solution to end up with a 40% saline solution. How much of the 25% solution will he need? How much will he end up with in total?

SEQUENCE NOTATION and ARITHMETIC SEQUENCES
A **sequence** may be referred to as "A_n". The **terms** of a **sequence** are usually named "a_n", usually with the subscripted letter "n" being the "index" or counter (the letters a and n are arbitrary, and can be represented by other letters). So the second term of a sequence might be named "a_2" (pronounced "ay-sub-two"), and "a_{12}" would designate the twelfth term.
The **common difference** of an **arithmetic sequence** is often referred to by the letter “d.”
The **explicit formula** for an **arithmetic sequence** can be written as an+b where a= the common difference, n= the term number, and b= the “zero” term. Similarly, it can be written a(n-1) + b where b=the first term.
Example: For the sequence: 3, 7, 11, 15, 19, … First Term: Common difference: Explicit Formula:
Example: For the sequence: a_n=2n+1 a_1: d: a_{25}:

SERIES NOTATION and ARITHMETIC SERIES

A **series** is where we add up some or all of the terms in a sequence.

A **partial sum** is when we choose to add up a specific number of terms of a sequence.

To indicate a **series**, we use the Greek letter Σ corresponding to the capital "S", which is called "sigma" (SIGG-muh)

To show the summation of the first through tenth terms of a sequence, we would write the following:

$$\sum_{n=1}^{10} a_n$$

The "n = 1" is the "lower index", telling us that "n" is the counter and that the counter starts at "1"; the "10" is the "upper index", telling us that a_{10} will be the last term added in this series; "a_n" stands for the terms that we'll be adding. The whole thing is pronounced as "the sum, from n equals one to ten, of a-sub-n". The summation symbol above means the following:

$$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 + a_{10}$$

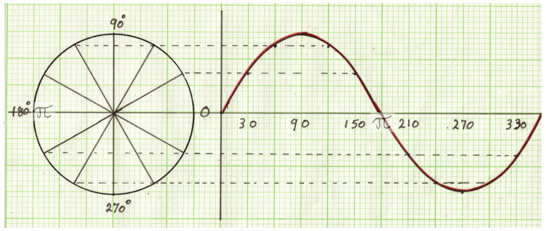
The written-out form above is called the "expanded" form of the series, in contrast with the more compact "sigma" notation.

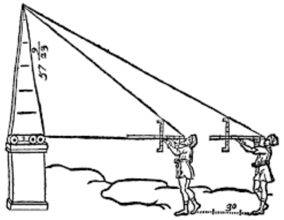
What formula can we use to find the sum of an arithmetic series?

Example: Write in expanded and in sigma notation the sum of the first 8 terms of the sequence a_n=4n-5

Arithmetic Sequences and Series Summary Notes	
<u>Arithmetic Sequences</u> **Example**: 5, 8, 11, 14, 17, … **General Term Equation:** $an + b$ *a*= Common difference *b*= "zero" term a_n= *n^{th} term* **Example:** First term = 5 Common difference = 3 Zero term = 5-3 = 2 Explicit Formula: $3n + 2$ $a_6 = 3n + 2$ $= 3(6)+2$ $= 20$	**<u>Arithmetic Series</u>** **Example:** 5 + 8 + 11 + 14 + 17 + … **Sigma notation (example):** $\sum_{n=1}^{20} 3n + 2$ **Partial Sum:** $S_n = \frac{n(a_1 + a_n)}{2}$ **Example:** $S_6 = \frac{20(5 + 62)}{2}$ $= 670$

Unit 3: Big Ideas





My notes:

Teacher notes:

- A scalar quantity has magnitude, and a vector quantity has both direction and magnitude.
- Vector addition can be used to solve problems from the world involving force and direction. There are several ways to solve these problems using trig or graphing.
- We can find all measurements in right triangles given either one side and one angle, or two side lengths.
- Equations for circles can be formed by using the Pythagorean Theorem and the relationships in right-angled triangles, and that Trig functions can be derived in this way.
- The graph of a sine functions has a characteristic shape and behavior, which is an excellent model for certain situations. (eg. hours of daylight over time, height of tides, etc.)
- Periodic functions make excellent models for many situations that fluctuate in non-linear ways. Examples are day length or tidal changes over time.

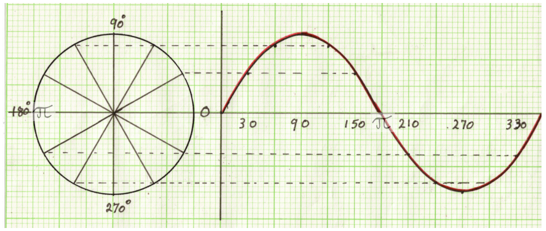
Unit 3: Trigonometry

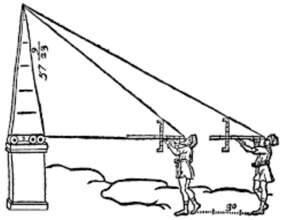
Unit 3: Trigonometry

Unit 3: Trigonometry

	Skills and Facts
1	I can recognize a situation that modulates and can be represented well with a sine function.
2	I can create and transform a graph of a sine function to match a situation. (eg. height of a ferris wheel over time.)
3	I can find missing side lengths of a right triangle given an angle and one side length or two side lengths
4	I can find missing angles of a right triangle given one angle and one side length or two side lengths
5	I can articulate the difference between a scalar and a vector
6	I can use right angle trigonometry to solve problems involving vector addition
7	I can use the graphing method (protractor and ruler) to solve vector addition problems

Unit 3: Big Ideas





My notes:

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Unit 3: Advanced Extensions

	Skills and Facts
1	I can solve complex problems involving vector addition
2	I can use my trigonometry skills to solve complex problems

Additional Notes

Unit 3: Trigonometry: Honors-Level Extensions

Unit 3: Advanced Extensions

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Additional Notes

Unit 3: Trigonometry: Honors-Level Extensions

Unit 3: Trigonometry

Essential Questions
What is the difference between a scalar and a vector?
How and why do we use vectors?
If we know the lengths & measures of SOME sides & angles of a triangle, when & how can we find all the others?
How are equations for circles related to right triangles?
What do the graphs of Trig functions look like and how do they behave?
What kinds of situations can be modeled by periodic functions?

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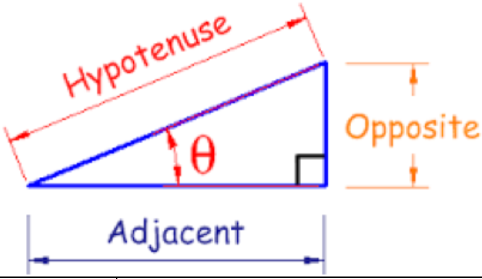
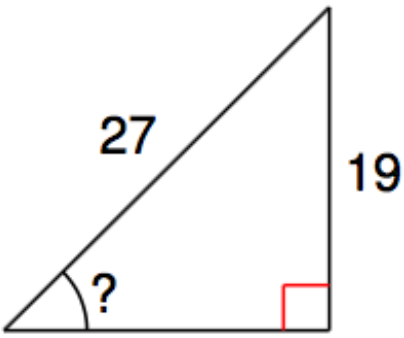
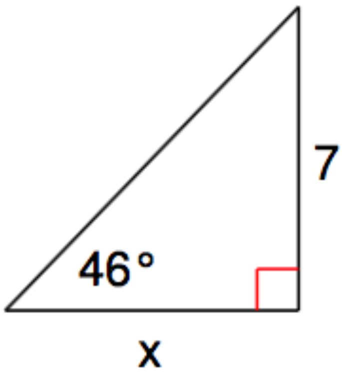
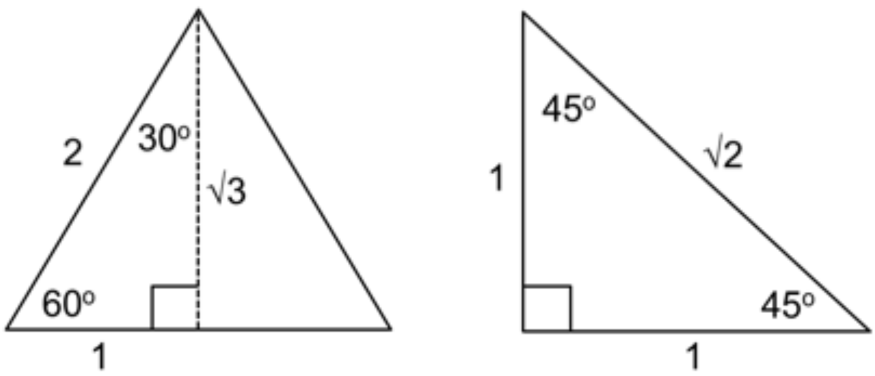
THREE TRIG FUNCTIONS: SOH CAH TOA		
The ratios of the three sides of right triangles can be used to find missing angles and side lengths. 		
SOH Sine=	CAH Cosine=	TOA Tangent=
What are some things that we do with these ratios?		
		
		

EXACT TRIG VALUES

The ratios of the three sides of right triangles can be used to find the exact values for some common trig ratios.

exact values in trigonometry



angle	sin	cos	tan
0°			
30°			
45°			
60°			
90°			

GRAPHING THE SINE FUNCTION

Write the general form for a sine function here:

The sine funcion can be used to effectively model many situations that modulate and repeat. This family of functions are called…

Sketch a graph of the parent function for sine: $f(x)) = \sin x$

Describe the transformations for the sine function, and identify which letter from the general form above makes each transformation:

1. Period: ____

Transformed by: _________

2. Amplitude: ____

Transformed by: _________

3. Vertical Shift: ____

Transformed by: _________

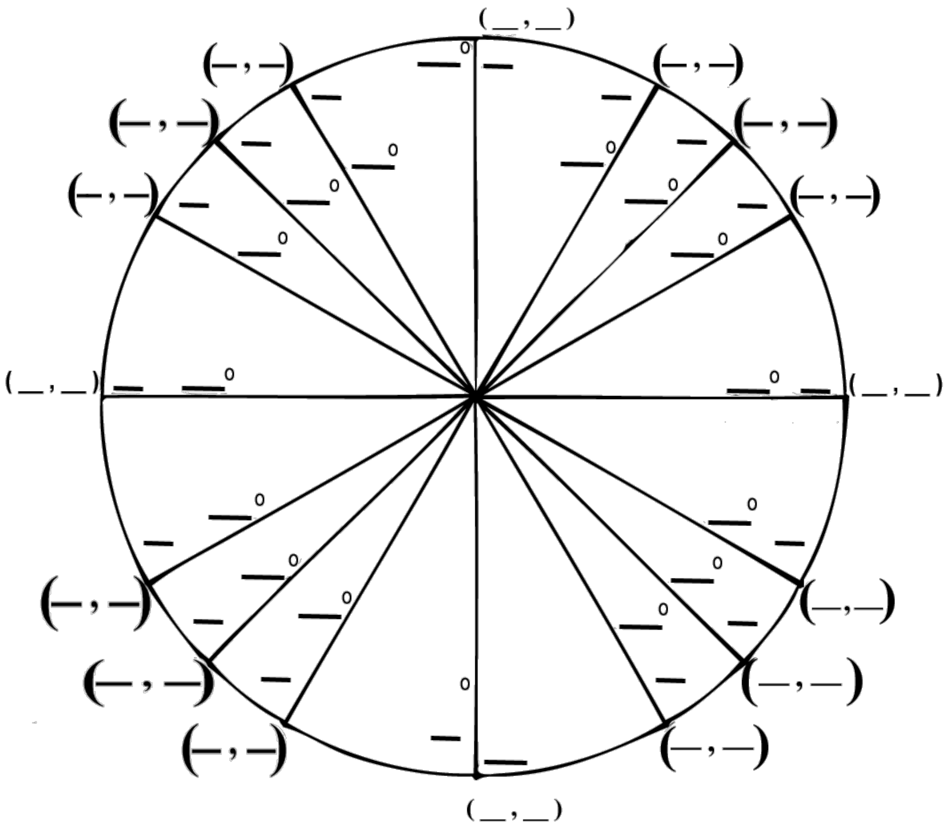
4. Horizontal Shift: ______________________________________

Transformed by: _________

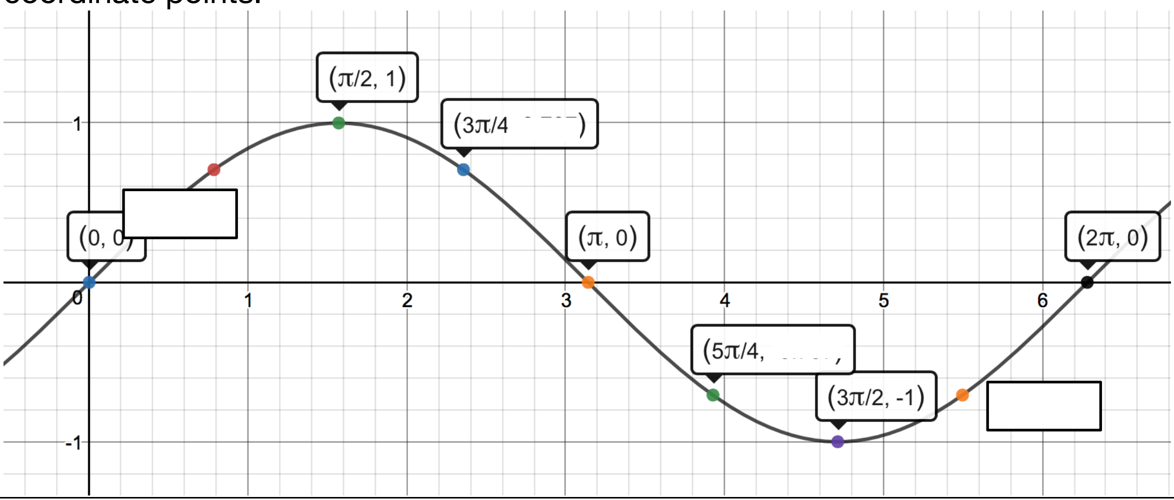
THE UNIT CIRCLE

The unit circle is a circle, which has a center on the origin (0,0), and has radius of one. Among other things, it is very useful for helping us to study trig functions!

Fill in degrees, radians, and the exact values of the coordinate points.



Notice how the unit circle relates to the Sine function, and fill in some more coordinate points!



VECTORS

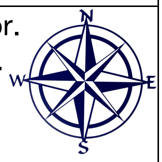
Scalars have ________________________

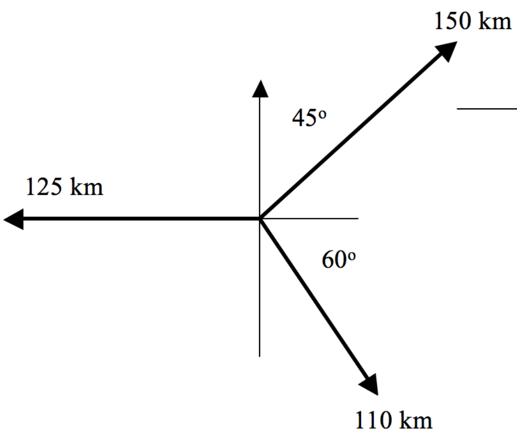
Example:

Vectors have both ____________________and ____________________

Example:

Notation: We need to indicate both the magnitude and direction of a vector. Label the following vectors, using the compass heading as your reference.





____

____

____

Adding and Subtracting Vectors:

When you add two vectors, the two you add are called _________________ vectors. The vector that results from adding two vectors is called the ______________ vector.

Always add vectors visually head to tail – move one of the vectors to the head or tail of the other if necessary.

Example: Add the following vectors:

Applying VECTORS

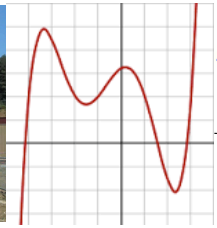
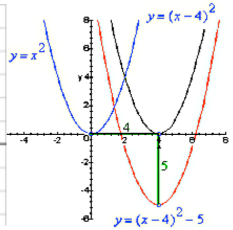
We can use Trigonometry or the “Graphing Method” to solve vector problems.

Example: A plane travels at 470 km/h in still air (no wind). The plane travels due north but is pushed sideways by an 85 km/h wind coming from 56° N of E. What is the resultant velocity and direction of the plane?

Method 1: Trigonometry	Method 2: Graphing
Make a sketch of the situation, and use trig to solve for the speed and direction of the resultant vector.	Make a careful drawing using a protractor and a ruler.

The resultant vector is:

Unit 4: Big Ideas

My notes:

Teacher notes:

1 Quadratic functions are distinguished by x^2 ;their graphs make a distinct shape called a parabola and quadratic equations are used to model situations in which one variable varies as the square of another.

2 There are many ways to solve a quadratic equation; the method chosen in a specific case depends on the information that is given and the preference of the mathematician. There will be one, two, or no real solutions/roots, which are the x-intercepts of the graph.

3 Equivalent representations of a function highlight different properties. We can use the tools of algebra to move between different forms. All graphs of quadratic functions are transformations of the parent function: $y=x^2$. All changes to graphs of functions through transformations (horizontal shifts, vertical shifts, or horizontal or vertical stretches) keep them in the same family. Any changes to the graph of a parent function ($y=x^2$ for quadratics) other than a horizontal shift, a vertical shift, or a horizontal or vertical stretch turn it into another kind of function.

4 Finding zeros of a polynomial allows us to use factoring to separate the components of the equation into simpler pieces.

5 Quadratic equations arise from problems involving areas of rectangles.

6 The square roots of negative numbers are pure imaginary numbers and all are multiples of $\sqrt{-1}$; i^2 *is* defined as -1. If there are no real solutions to a quadratic, there are solutions, using i on the set of complex numbers.

7 Quadratic functions commonly appear in the world. They model many real world phenomena including projectile motion

8 Many of the same techniques we use for analyzing quadratics (factoring, transformation of graphs) can be applied to all polynomials.

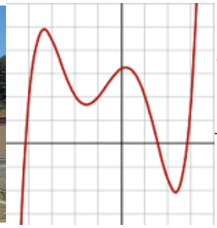
Unit 4: Quadratics and Polynomials

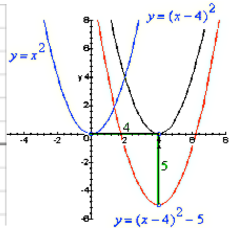
Unit 4: Quadratics and Polynomials

Unit 4: Quadratics and Polynomials

	Skills and Facts
1	I can solve quadratic equations by factoring, completing the square, and use of the quadratic formula
2	I can sketch graphs and label significant parts of quadratics when given an equation, and write equations given a quadratic graph. I can transform a graph from a parent function.
3	I can use technology (eg. Desmos, TI Calculator, Geogebra) to create graphs of quadratics, and to perform transformations on quadratic equations
4	I can recognize and apply properties of perfect square trinomials.
5	I can create a quadratic math model and solve problems given a situation (eg. when will the ball hit the ground? …How high will the rocket fly? …When do you make the most profit?)
6	Use algebra to move flexibly between different forms of equations eg. multiplying binomials to get trinomials, move from vertex to standard form of a quadratic, etc.
7	I can add, subtract, multiply, divide, and fully factor polynomial expressions
8	I can collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models.

Unit 4: Big Ideas





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Unit 4: Advanced Extensions

	Skills and Facts
1	I can describe end behavior and zeros of polynomials
2	Perform operations on complex numbers, express the results in simplest form using patterns of the powers of i, and identify solutions to quadratic equations that are valid for the complex numbers.
3	I can use technology (eg. Desmos, TI Calculator, Geogebra) to create graphs of cubics, and higher degree polynomials
4	Solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically.

Additional Notes

Unit 4: Quadratics: Honors-Level Extensions

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Additional Notes

Unit 4: Quadratics: Honors-Level Extensions

Unit 4 Vocabulary	
Quadratic	A **mathematical expression** containing a term of the **second degree**, such as $\mathbf{x^2 + 2}$
Polynomial	A **mathematical expression** consisting of a **sum of terms**, each term including a **variable** or variables raised to a **power** and multiplied by a **coefficient**. In Algebra 2, we typically refer to polynomials of degree greater than 2 to distinguish them from quadratics.
Quadratic Formula	A method of **solving quadratic equations**, which has been **derived** by **completing the square** for a quadratic in the form: $\boldsymbol{ax^2 + bx + c = 0}$
Complete the square	A technique used to **solve quadratic equations** and **graph quadratic functions.** Also known as the "box" method.
Parabola	The characteristic shape formed by the **graph of a quadratic function**.
Vertex	The point where the **parabola** crosses its axis of symmetry
Discriminant	The part of the quadratic equation "inside" the square root. The discriminant can tell us if a quadratic has zero, one, or two real solutions.
Line of Symmetry	In the graph of a quadratic function, the line of symmetry is a vertical line that divides the parabola exactly in half. If the line of symmetry were a mirror, the reflection would be exactly
Degree	For a **polynomial** with one **variable** the **degree** is the largest **exponent** of that variable
Root (Zero)	A **solution** to an **equation** or the place where the **graph** of a **function** crosses the **x-axis**; where **y=0**.

Unit 4 Vocabulary	
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QUADRATIC EQUATIONS: SOLVING BY THE BOX METHOD

The box method is also called:

There will be __________, __________, or __________ real solutions.

Level 0: Take the positive and negative square root of both sides to get two solutions:

$\boldsymbol{x^2 = 100}$ $\boldsymbol{x^2 = 17}$

Level 1:

$\boldsymbol{(x-2)^2 = 25}$

- Take the square root of both sides
- Solve for x

Level 2:

$\boldsymbol{x^2 + 6x + 9 = 36}$

- Use the box to make an area model to rewrite the trinomial as a perfect square binomial
- Take the square root of both sides
- Solve for x

QUADRATIC EQUATIONS: SOLVING BY THE BOX METHOD: : LEVEL 3

Level 3:

$\boldsymbol{x^2 + 4x + 15 = 111}$

- Use the box to make an area model to rewrite the trinomial as a perfect square binomial
- Compare the bottom right box to the constant; add or subtract to make them match (make sure to keep the equation balanced. You need to do the same thing to both sides of the equation)
- Take the square root of both sides
- Solve for x

QUADRATIC EQUATIONS: SOLVING BY THE BOX METHOD: LEVEL 4

Level 4:

$\boldsymbol{x^2 + 3x - 6 = 4}$ (Notice that the coefficient of the linear term is odd)

- Multiply through the whole equation by 4
- Use the box to make an area model to rewrite the trinomial as a perfect square binomial
- Compare the bottom right box to the constant; add or subtract to make them match (make sure to keep the equation balanced. You need to do the same thing to both sides of the equation)
- Take the square root of both sides
- Solve for x

QUADRATIC EQUATIONS: SOLVING BY THE BOX METHOD: LEVEL 5!

Start with a quadratic equation in the form $ax^2 + bx + c = d$

Steps:

- Multiply through by *a* (to make sure you have a perfect square x^2 term
- Multiply through by 4 (to make the coefficient of the linear term even)
- BOX it!
- Compare the constant to your original equation; add or subtract to make them match (make sure to keep the equation balanced. You need to do the same thing to both sides of the equation)
- Rewrite your trinomial as a perfect square binomial
- Take the square root of both sides
- Solve for x

Example:

$3x^2 - 8x - 6 = 5$

QUADRATIC EQUATIONS: Deriving the quadratic formula

Start with a quadratic equation in the form $ax^2 + bx + c = 0$

Steps: Treat this like a regular level 5 quadratic!

1. Multiply through by *a* (to make sure you have a perfect square x^2 term
2. Multiply through by 4 (to make the coefficient of the linear term even)
3. BOX it!
4. Compare the constant to your original equation; add or subtract to make them match (make sure to keep the equation balanced. You need to do the same thing to both sides of the equation)
5. Rewrite your trinomial as a perfect square binomial
6. Take the square root of both sides
7. Solve for x

$ax^2 + bx + c = 0$

QUADRATIC EQUATIONS: Using the quadratic formula

Start with a quadratic equation in the form $ax^2 + bx + c = 0$

Remember to set your quadratic equal to zero or this won't work!

- Identify a, b, and c
- Substitute into the quadratic formula to find your solutions
- Be super careful with your arithmetic

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example:

$2x^2 + 2x - 17 = -5$

a=

b=

c=

QUADRATIC EQUATIONS: Using the discriminant	
Formula: $$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$	Discriminant:

The discriminant can tell us how many real solutions there are to a quadratic equation.

If the discriminant is positive, there are ______ solutions.

If the discriminant is negative, there are ______ solutions.

If the discriminant is zero, there are _________ solutions.

Example: Calculate the discriminant to find out how many real solutions there are.

$3x^2 - 2x + 5 = -12$	$2x^2 - 3x + 2 = 0$	$25x^2 - 20x - 64 = -10$

Factoring: Identities
$(x+y)^2 =$ Example: $(x+5)^2 =$
$(x-y)^2 =$ Example: $(x-6)^2 =$
$x^2 - y^2 =$ Example: $x^2 - 25 =$ Example 2: $81x^2 - 4 =$
$(x^n + x^{n-1} + x^{n-2} + \cdots + x + 1)(x - 1\ ) =$ Example: $(x^4 + x^3 + x^2 + x + 1)(x - 1\ ) =$

Factoring: Identities
$(x+y)^2 =$ Example: $(x+5)^2 =$
$(x-y)^2 =$ Example: $(x-6)^2 =$
$x^2 - y^2 =$ Example: $x^2 - 25 =$ Example 2: $81x^2 - 4 =$

Factoring Trinomials when a>1

Fully factor the following trinomial $15x^2 - 27x - 6$

Step 1: Check to see if there is a common factor that you can take out.

Step 2: Multiply $a * c$

Step 3: List the factors of ac

Step 4: Check to see if you have a pair that add to b

Step 5: Use the reverse box method to fill the boxes

Step 6: Find the Greatest Common Factor of each row and column

Step 7: Rewrite your factors as multiplication

Factor when “a” does not equal 1

Let’s begin with this example:

$2x^2 + x - 6$

The first step in factoring will be to multiply "a" and "c"; then we'll need to find factors of the product "ac" that add up to "b".

a=

b=

c=

…so ac= (   )(   ) = ______

So we need to find factors of _________ that add up to _________

List the factor pairs, and circle the one that works:

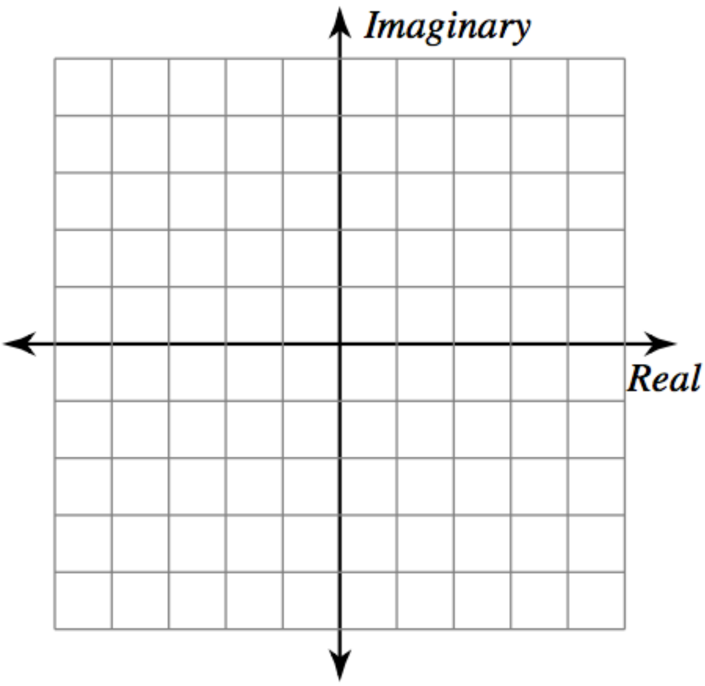
Use the area model (aka the box method)

Then re-write the side lengths in factored form!

(   ) (   )

Complex Numbers Intro
We have come across situations with quadratics where we end up with a negative number inside the square root. Until now, we have always said "impossible!" and moved on. But no more! Now we have invented a way to deal with this situation.
What is the definition of i^2

- Complex numbers are written in the form $a + bi$ where a and b are real numbers, and i represents the positive or negative root of -1.
- We can graph complex numbers by counting the real part on a horizontal axis (real numbers), and counting the imaginary part on a vertical axis (imaginary numbers).



Examples:

$2 + 3i$

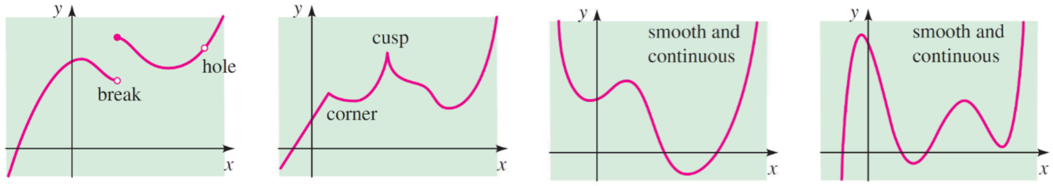
$-1 + 2i$

$4 - i$

POLYNOMIALS

Graphs of Polynomials

The graph of a polynomial function is always a smooth curve; that is, it has no breaks or corners.



Not the graph of a polynomial function | Not the graph of a polynomial function | Graph of a polynomial function | Graph of a polynomial function

End Behavior:

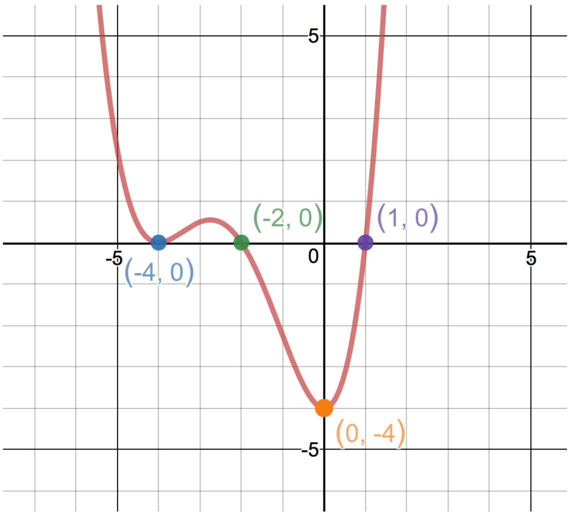
The graph of a polynomial of odd degree has the following end behavior:	The graph of a polynomial of even degree has the following end behavior:

$$a(x+b)^m(x+c)^n(x+d)\ \dots$$

Factored form helps us to write equations for a polynomial, because:

We use the zero-product property to locate the roots, and then substitute the coordinates of another point to find the "a" value.

Example:



QUADRATIC Features: How to…					
These notes will help you compare characteristics of quadratic functions. In the table below, fill in the missing entries.					
	Find the Roots	**Find the Minimum / Maximum**	**Find the Axis of Symmetry**	**Find the Opening direction**	**Find the y-intercept**
Given a graph	Look for where it crosses the x-axis	Look for the vertex	Look for the vertex	Just look at it!	Look for where it crosses the y-axis
Given an equation in Vertex Form: $\boldsymbol{f(x) = a(x-h)^2 + k}$					
Given an equation in standard form: $f(x)) = ax^2 + bx + c$					
Given an equation in factored form: $\boldsymbol{f(x)) = a(x+b)(x+c)}$					

QUADRATIC Features: How to…

These notes will help you compare characteristics of quadratic functions. In the table below, fill in the missing entries.

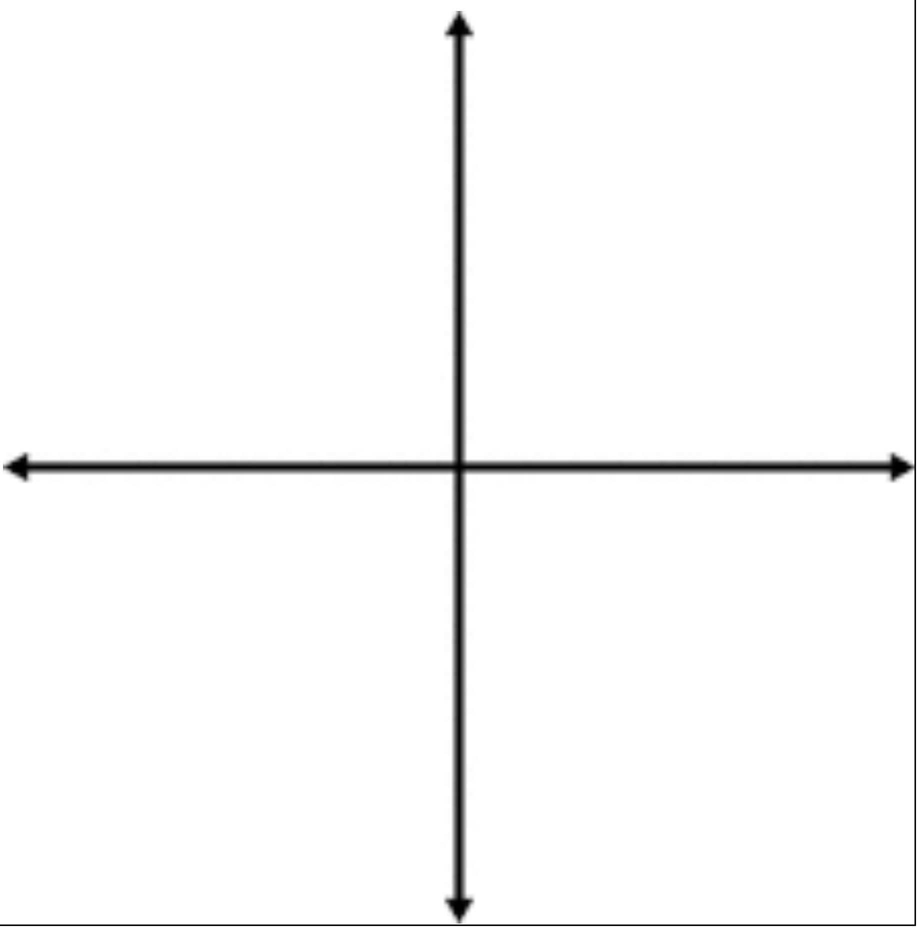
	Find the Roots	**Find the Minimum / Maximum**	**Find the Axis of Symmetry**	**Find the Opening direction**	**Find the y-intercept**
Given a graph	Look for where it crosses the x-axis	Look for the vertex	Look for the vertex	Just look at it!	Look for where it crosses the y-axis
Given an equation in Vertex Form: $\boldsymbol{f(x) = a(x-h)^2 + k}$	Solve for x	????	????	????	????
Given an equation in other form	Convert it into standard form, then use the quadratic formula	Convert it into standard form, then ????	Convert it into standard form, then ????	Convert it into standard form, then ????	Plug in $x = 0$

QUADRATICS – Find the features

Given the quadratic equation in standard form:

$$y = x^2 + 2x - 15$$

1. Find the vertex: _______
2. Find the line of symmetry: ______________
3. Rewrite in vertex form :_______________
4. Sketch the graph
5. Find the x-intercepts: _______________
6. Find the y-intercept: ________________
7. State the minimum or maximum: ______________



QUADRATIC SEQUENCES
We can use the method of finite differences to determine if a sequence is behaving like a polynomial. If we find that a sequence has a common difference (1st, 2nd, 3rd, etc.) we can use the method of finite differences to work out an explicit formula for the n^{th} term.

EXAMPLE: 5, 9, 14, 20, 27, 35, …

5

9

14

20

27

35

Since there is a common second difference, we know that this is a

________________________ Sequence. We can compare the numbers

from our chart to the finite differences chart to find values for a, b, and

c, and use them to write an explicit formula for this sequence:

DERIVING THE QUADRATIC SEQUENCE FORMULA

We know that all quadratic sequences can be written in the form $\boldsymbol{an^2 + bn + c}$ Starting from this assumption, substitute the term numbers in for n to create a chart showing the second differences.

***note:** *leave the grey boxes blank.*

Term Number	Sequence	First Difference	Second Difference
0			
1	$a + b + c$		
2	$4a + 2b + c$		
3	$9a + 3b + c$		
4			
5			
6			

QUADRATIC SEQUENCES: Method Two

We know that all quadratic sequences can be written in the form $\boldsymbol{an^2 + bn + c}$ Starting from this assumption, We can break apart the sequence into one sequence: $\boldsymbol{an^2}$ added to another sequence: $\boldsymbol{bn + c}$

EXAMPLE: Given the sequence: 3, 13, 27, 45, 67, 93, …
Write an explicit formula for the n^{th} term.

Step 1. Find the consistent second difference.

3

13

27

45

67

93

Step 2: Divide the second difference by 2 to find **a**: ___________

Step 3: Subtract $\boldsymbol{an^2}$ from each term to yield a linear (arithmetic) sequence:

1, 5, 9, 13, 17, 21, …

Step 4: Find the explicit formula for the linear sequence.

Step 5: Add the two together.

Quadratic Graphing

Vertex Form of a QUADRATIC **Equation**:

How do the constants change the graph of the parent function $f(x) = x^2$

a: ______________________________

h: ______________________________

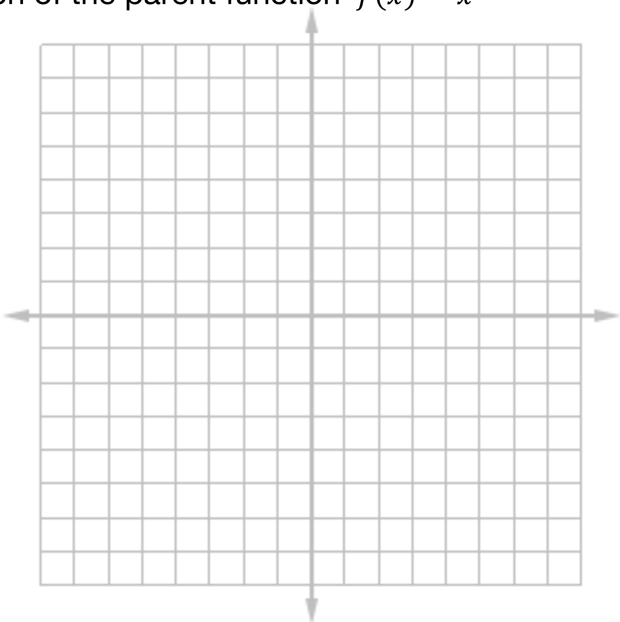
k: ______________________________

If you have an quadratic equation in vertex form, what are the coordinates of the **turning point** (the "vertex") of the graph?

Example: for the equation $f(x) = \frac{1}{2}(x-3)^2 + 8$

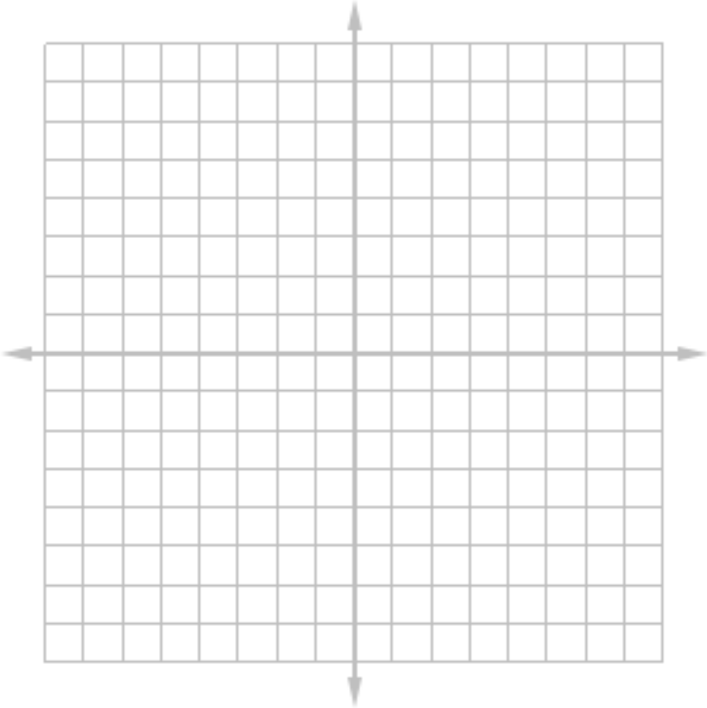
The location of the turning point (vertex) is (,)

Sketch a graph of the parent function $f(x) = x^2$

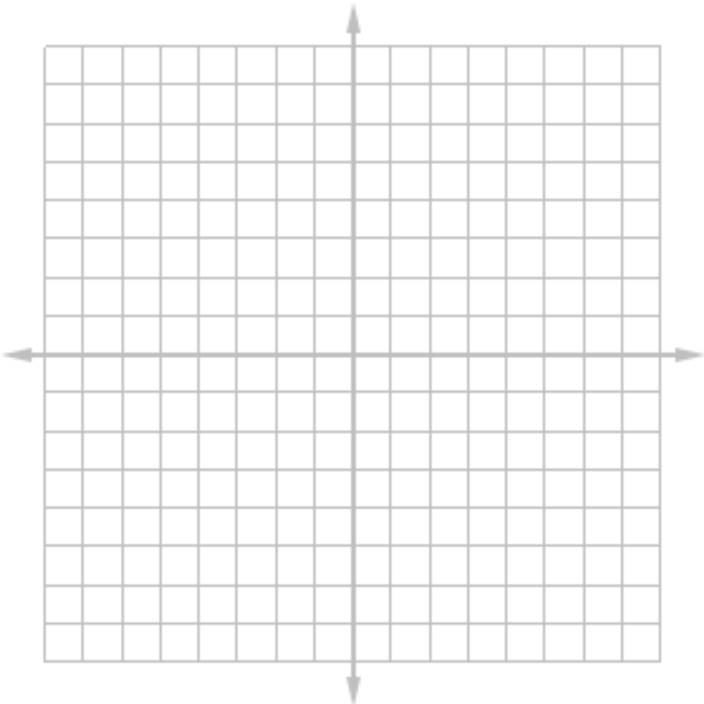


QUADRATICS

Sketch a graph of the function $f(x) = x^2 - 3$



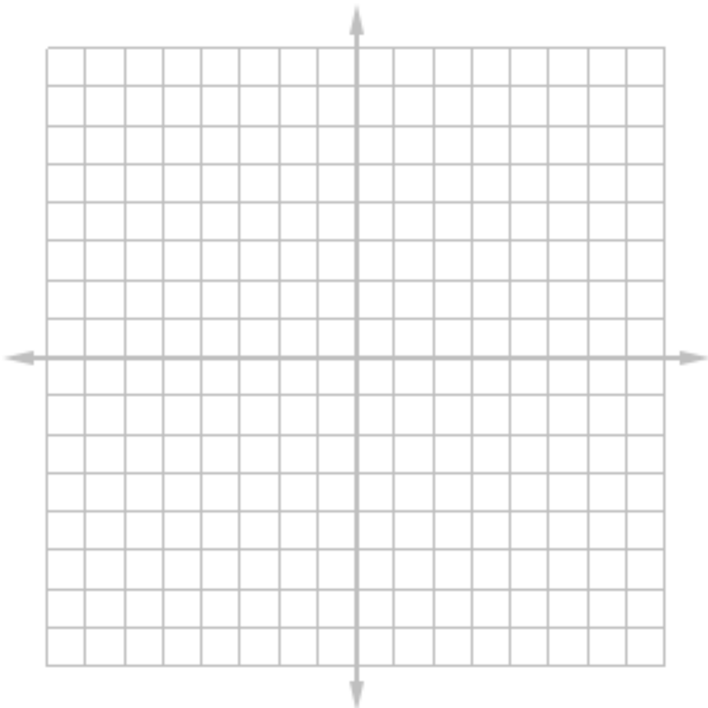
Sketch a graph of the function $f(x) = -(x-3)^2 + 1$



Quadratic Graphing by Factoring

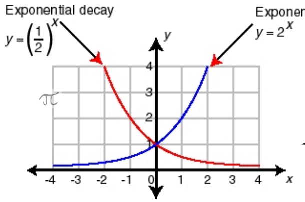
Sketch a graph of the function $f(x) = x^2 + 5x - 3$

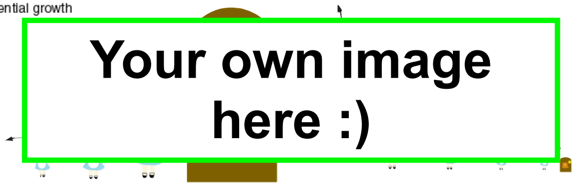
1. Take the GCF of the first two terms
2. Set each of the factors equal to zero to find two symmetrical y-values.
3. Find the line of symmetry
4. Plug in the x value to find the vertex



Quadratics with imaginary roots
Now that we can deal with $\sqrt{-1}$ let's practice with a quadratic equation that has no real solutions. Solve using the method of your choice, and include the imaginary solutions.
$11x^2 - 10x + 4 = -3$

Unit 5: Big Ideas





My notes:

Teacher notes:

- There are rules and procedures for arithmetic with exponential expressions which can save us from doing repeated or redundant calculations
- Geometric sequences relate to and behave like exponential functions, and can be used to model real situations. Geometric and Arithmetic sequences have specific similarities and differences.
- Exponential growth and decay have characteristic shape and behavior, and are substantially different from linear growth. Exponential graphs have a characteristic shape, and all exponential graphs are transformations from the parent function $y=b^x$.
- Roots can be expressed as rational exponents.
- Changing the form of expressions and equations is a way to communicate with specific mathematical grammar. Equations can be manipulated from one form to another, and different but equivalent forms reveal different aspects of a function

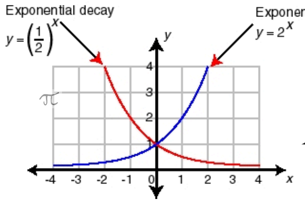
Unit 5: Exponential Functions

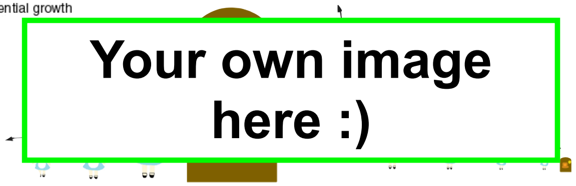
Unit 5: Exponential Functions

Unit 5: Exponential Functions

	Skills and Facts
1	• I can manipulate exponential equations by using the laws of exponents and roots, including the multiplication rule, the quotient rule, and the power of a power rule.
2	• I can rewrite a radical function or expression as an equivalent power function or expression.
3	• I can represent exponential functions numerically, algebraically, and graphically.
4	• I can write equivalent expressions involving radicals and exponents, including negative exponents.
5	• I can evaluate expressions involving radicals and exponents.
6	• I can sketch graphs of exponential functions and create graphs using technology
7	• I can write exponential equations to model a situation
8	• I can solve exponential equations for a given variable
9	• I can write an n^{th} term formula for a geometric sequence

Unit 5: Big Ideas





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Unit 5: Exponential Functions

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Unit 5: Advanced Extensions

	Skills and Facts
1	I can solve radical equations graphically and algebraically, and check for extraneous roots.
2	I can derive and use the formula to find the sum of a finite geometric series
3	I can rationalize the denominator of an expression

Additional Notes

Unit 5: Exponential Functions

Unit 5: Advanced Extensions

	Skills and Facts
1	I can solve radical equations graphically and algebraically, and check for extraneous roots.
2	I can derive and use the formula to find the sum of a finite geometric series
3	I can rationalize the denominator of an expression

Additional Notes

Unit 5: Exponential Functions

Unit 5: Exponentials

Essential Questions

- Why is it helpful to learn about the mechanics of exponents, and how do exponents work?
- How are geometric sequences related to exponential functions, and where do they show up in the world? How are they similar or different from arithmetic sequences and series?
- What does exponential growth or decay look like, and how does it compare to linear growth? How can graphs help us to visualize these differences?
- How are expressions involving radicals and exponents related?

Unit 5: Exponentials

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- How are expressions involving radicals and exponents related?

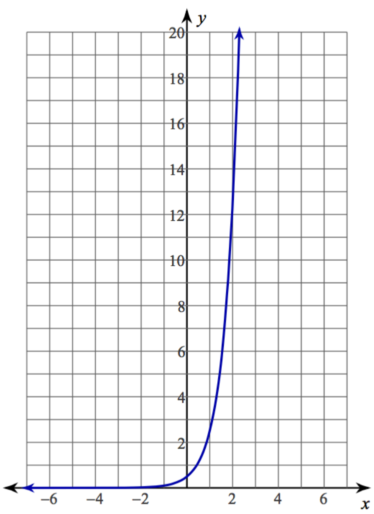
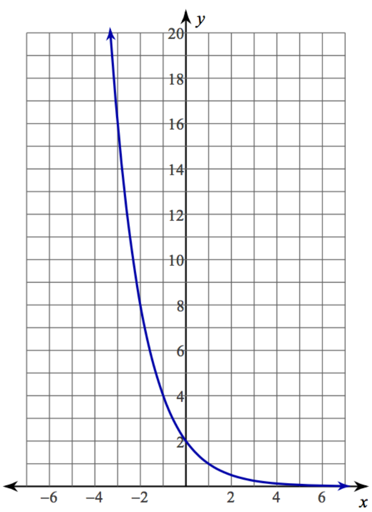
EXPONENTIAL GROWTH/DECAY

If an exponential function gets bigger and bigger, we call that:	If an exponential function gets smaller and smaller, we call that:

Given an exponential equation in the form $y = ab^x$ how can we tell if the equation represents exponential growth or decay?

Growth	Decay
When $a > 0$	When $a > 0$
and	and

Label these four examples as growth or decay

$y = 4 * 0.5^x$	$y = 6 * 3^x$
	

EXPONENTS: FRACTIONAL POWER RULES		
Fractional Powers	An exponent of $^1/_2$ is actually **square root** An exponent of $^1/_3$ is **cube root** An exponent of $^1/_4$ is **4th root** And so on!	$4^{\frac{1}{2}} = \sqrt{4}$ $4^{\frac{1}{3}} = \sqrt[3]{4}$ $4^{\frac{1}{4}} = \sqrt[4]{4}$ etc...

But WHY?

Remember the product rule: $x^2 * x^2 =$ __________

Try that with a fractional exponent: $9^{\frac{1}{2}} * 9^{\frac{1}{2}} =$ __________

What do we call a number that, when multiplied by itself, gives another number?

The square root! **So…$\mathbf{9^{\frac{1}{2}} = \sqrt{9}}$**

Example: Evaluate the following expressions:

$144^{\frac{1}{2}}$	$8^{\frac{2}{3}}$	$25^{\frac{3}{2}}$

EXPONENTS: POWER RULES	
POWER OF A POWER RULE: To raise a power to a power, multiply the exponents.	**POWER OF A PRODUCT RULE:** Each base is raised to the power
RULE: $(x^n)^m =$ __________________	**RULE:** $(xy)^m =$ ____________________
Example: $(4^2)^6 =$	Example: $(4 * 5)^6 =$
Example: $(5^3)^{11} =$	Example: $(5x)^3 =$
Example: $(8^a)^4 =$	Example: $(8x)^a =$

EXPONENTS: PRODUCT AND QUOTIENT RULES
PRODUCT RULE: When multiplying exponents with the same base, keep the base and add the powers. **RULE:** $x^n * x^m =$ ________________ Example: $4^2 * 4^4 =$ ________________ Example: $5^{11} * 5^4 =$ ________________ Example: $8^a * 8^4 =$ ________________
QUOTIENT RULE: When dividing exponents with the same base, keep the base and subtract the powers. **RULE:** $\frac{x^n}{x^m} =$ ________________ Example: $\frac{4^9}{4^6} =$ ________________ Example: $\frac{5^{11}}{5^4} =$ ________________ Example: $\frac{8^a}{8^4} =$ ________________

Geometric Sequences
Refresher: A **sequence** may be referred to by a letter as in "A". The **terms** of a **sequence** are named "a_n", usually with the subscripted letter "n" being the "index" or counter (the letters a and n are arbitrary, and can be represented by other letters). So the second term of a sequence might be named "a_2" (pronounced "ay-sub-two"), and "a_{12}" would designate the twelfth term.
The **common ratio** of a **geometric sequence** is often referred to by the letter "r."
• The **explicit formula** for a **geometric sequence** can be written as $a * r^{(n-1)}$ where a= the first term, *r*= the common ratio, *n*= the term number. • Similarly, it can be written $a * r^{(n)}$where a =the zero term.
Example: For the sequence: 7, 14, 28, 56, 112, … First Term: Common ratio: Explicit Formula:
Example: For the sequence: $a_n = \frac{1}{2} * 3^{n-1}$ $a_1 =$ $r =$ a_6=

SERIES NOTATION and GEOMETRIC SERIES
Refresher: A **series** is where we add up some or all of the terms in a sequence.
A **partial sum** is when we choose to add up a specific number of terms of a sequence.
Refresher: To indicate a **series**, we use the Greek letter Σ corresponding to the capital "S", which is called "sigma" (SIGG-muh) To show the summation of the first through sixth terms of a sequence, we would write the following: $$\sum_{n=1}^{6} 3 * \left(\frac{2}{3}\right)^{n-1}$$ The "*n* = 1" is the "lower index", telling us that "*n*" is the counter and that the counter starts at "1"; the "6" is the "upper index", telling us that a_6 will be the last term added in this series; The summation symbol above means the following: $$a_1 + a_2 + a_3 + a_4 + a_5 + a_6$$ The written-out form above is called the "expanded" form of the series, in contrast with the more compact "sigma" notation.
What formula can we use to find the sum of a **geometric series**?
Example: Write in expanded and in sigma notation the sum of the first 8 terms of the sequence $a_n = 4 * 2^{n-1}$

CALCULATING THE SUM OF A GEOMETRIC SERIES	
Use the geometric series formula to calculate the sum *S= the sum of the series* *a=the first term* *r=the common ration*	$S = a\left(\frac{1-r^n}{1-r}\right)$
Example: Evaluate the following series $$\sum_{n=1}^{6} 2^{n-1}$$	
Example: Evaluate the following series $$\sum_{n=1}^{6} 3 * \left(\frac{2}{3}\right)^{n-1}$$	

Geometric Sequences and Series Summary Notes

Geometric Sequences

Example:
5, 15, 45, 135, 405, …

General Term Equation:
$a * r^{(n-1)}$
a= First Term
r= common ratio
a_n = n*th term*

Example:
First term = 5
Common ratio = 3
Explicit Formula: $5 * 3^{n-1}$

$a_6 = 5 * 3^{6-1}$
$= 5 * 243$
= 1215

Geometric Series

Example:
3 + 6 + 12 + 24 + 48 + 96

Sigma notation (example):

$$\sum_{n=1}^{6} 3 * 2^n$$

Partial Sum:

$$S_n = a\left(\frac{1 - r^n}{1 - r}\right)$$

Example:

$$S_6 = 3\left(\frac{1 - 2^6}{1 - 2}\right)$$

$$= 3\left(\frac{1-64}{-1}\right) = 3 * 63 = 189$$

$$\sum_{k=0}^{n-1}(ar^k) = a\left(\frac{1-r^n}{1-r}\right)$$

a is the first term

r is the **"common ratio"** between terms

n is the number of terms

Infinite Geometric Series

Is there some way that we can think about what happens when we try to calculate the sum of an infinite series? Is it possible that by adding an infinite collection of terms that we can come to a finite solution?

Consider the following series:

$$\sum_{n=1}^{\infty} 3 * (2)^{n-1}$$

The terms in the sequence are: 3, 6, 12, 24, …
We can see that each term gets bigger and bigger without limit, and that the sum of this series just gets bigger and bigger the more terms we add. This series is said to **diverge.**

Now consider this series:

$$\sum_{n=1}^{\infty} 0.9 * (0.1)^{n-1}$$

The terms in the sequence are: 0.9, 0.09, 0.009, 0.0009, …
Each term gets smaller and smaller, so if we try to evaluate the sum of more and more terms, the number we are adding gets closer and closer to zero. This series is said to **converge.**

You can take the sum of an *infinite* geometric sequence, but only in the special circumstance that the common ratio *r* is between –1and 1; that is, you have to have | *r* | < 1. Notice in our geometric series formula, that when | *r* | < 1, the r^npart gets smaller and smaller – in fact it becomes so small that it is so close to zero that it loses meaning and importance (…such a sad ratio – but it lead to something exciting, so it's all OK!) – so we can just take it out of the formula!

In the special case that $|r| < 1$, the infinite sum exists and has the following value:

$$\sum_{i=1}^{\infty} a_i = \frac{a}{1-r}$$

In our class, you will be responsible for checking to see whether a series **converges** or **diverges**.

Example: Does the infinite series: 7, 14, 28, 56, 112, … converge or diverge?

The common ratio is: _______

So we know that the sum of the infinite series ________________________________

Example: Does the infinite series: 48, 24, 12, 6, 3, … converge or diverge?

The common ratio is: _______

So we know that the sum of the infinite series ________________________________

Working with RADICALS

Remember: a radical expression (radical is the same thing as root)is the same thing as an exponential expression, just written in a different way. Draw arrows in to your diagram to indicate where the parts are written in each form.

$$\sqrt[\square]{x^{\square}} = x^{\frac{\square}{\square}}$$

Combining like terms: You can combine like terms with radical expressions in the same way that you do with variables.

Example: $2\sqrt{3} + 7\sqrt{3} =$	**Example:** $12\sqrt{2} - 7\sqrt{2} =$

Simplifying Radicals: There are two things to do to make sure that your radical expression is "simplified."

1. Take everything out of the radical that you can
2. "Rationalize" the denominator (get rid of any radicals in the denominator).

Example: simplify the expression $\sqrt{\mathbf{2}} + \sqrt{\mathbf{32}}$	**Example:** simplify the expression $\sqrt{\mathbf{50}} + \sqrt{\mathbf{125}}$
Example: simplify the expression $\frac{\sqrt{\mathbf{2}}}{\sqrt{\mathbf{3}}}$	**Example:** simplify the expression $\frac{\mathbf{2}\sqrt{\mathbf{12}}}{\mathbf{2}\sqrt{\mathbf{15}}}$

Writing Equations for EXPONENTIAL FUNCTIONS

We can write equations for exponential functions in the form:

$$y = a * b^{x-h} + k$$

The "a" represents: The initial condition or *Where you start*

The "b" represents the ____

On the graph of an exponential:

a: ____

h: ____

k: ____

Example:

x	0	1	2	3	4	5	6	7
y	6	18	54	162	486	1458	4374	13122

a=

b=

Equation:

Example:

x	0	1	2	3	4	5
y	63	21	7	$\frac{7}{3}$	$\frac{7}{9}$	$\frac{7}{27}$

Writing Equations for EXPONENTIAL SITUATIONS

Use the form $y = ab^x$ to write an equation for an exponential situation

The bacteria *E. coli* often cause illness among people who eat infected food. Suppose that a single *E. coli* bacterium in a batch of ground beef begins doubling every minute.

a. How many bacteria will there be after 1, 2, 3, 4, and 5 minutes have elapsed? (Assume no bacteria die.)

0 minutes:

1 minute:

2 minutes:

3 minutes:

4 minutes:

5 minutes:

b. Write an equation that can be used to calculate the number of bacteria in the food after any number of minutes.

Let x = # of minutes. y = # of bacteria. a=Initial condition. b=Growth factor

a=

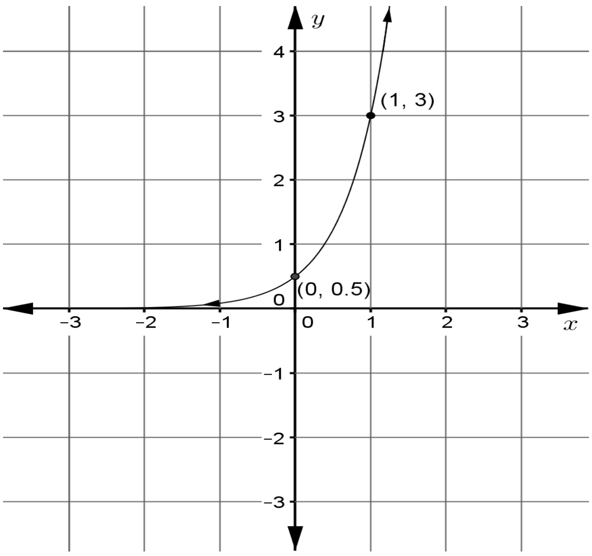
b=

Equation:

y =

Writing Equations from GRAPHS for EXPONENTIAL FUNCTIONS

The graph of an exponential function is shown below.



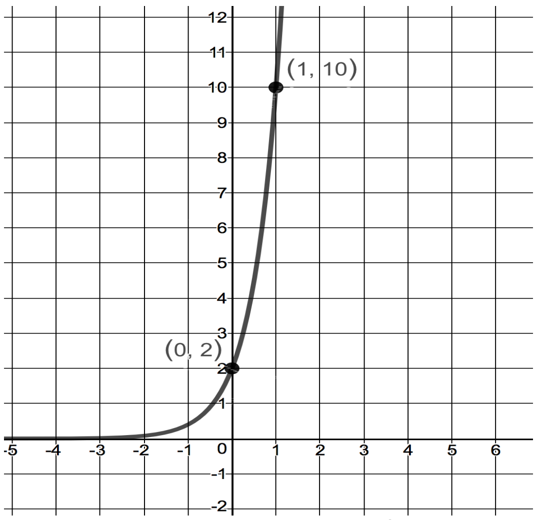
Write an equation that represents this function

The Initial Condition or y-intercept (*a*):___________

The Growth Factor (*b*): _________

Substitute your values into $f(x) = ab^x$ to write the equation:

Example:



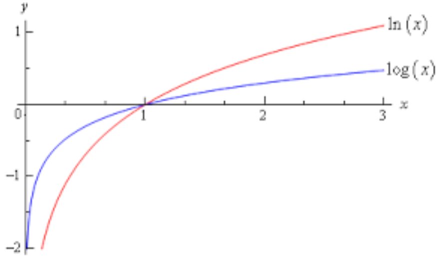
Write an equation that represents this function

The Initial Condition or y-intercept (*a*):___________

The Growth Factor (*b*): _________

Equation:

Unit 6: Big Ideas



$$\begin{array}{lll} \log_5 125 = 3 & \text{since } 5^3 = 125 \\ \log_3 81 = 4 & \text{since } 3^4 = 81 \\ \log_2 32 = 5 & \text{since } 2^5 = 32 \end{array}$$

My notes:

- Log functions are inverses of exponential functions; an inverse function is a function that “undoes” another function; if f(x) maps x to y, then $f^{-1}(x)$ maps y back to x.
- We can rewrite the laws of exponents to work with logarithms and solve logarithmic equations.
- Logarithm laws often let us change multiplication into addition. This leads to much easier methods for solving problems.
- The graph of a log function has a characteristic shape and behavior, which is an excellent model for certain situations.

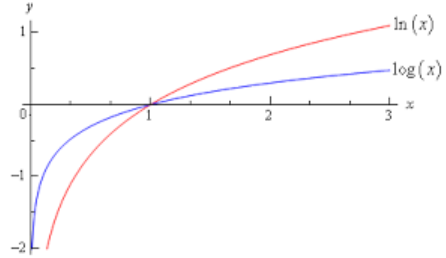
Unit 6: Logarithmic Functions

Unit 6: Logarithmic Functions

Unit 6: Logarithmic Functions

	Skills and Facts
1	• I can rewrite an exponential equation as a log, and a log equation as an exponential.
2	• I can represent logarithmic functions numerically, algebraically, and graphically.
3	• I can evaluate expressions involving logarithms.
4	• I can sketch graphs of logarithmic functions and create graphs using technology
5	• I can write logarithmic equations to model a situation
6	• I can solve logarithmic equations for a given variable
7	• I can use the laws of logarithmic to simplify logarithmic equations

Unit 6: Big Ideas



$$\log_5 125 = 3 \quad \text{since } 5^3 = 125$$
$$\log_3 81 = 4 \quad \text{since } 3^4 = 81$$
$$\log_2 32 = 5 \quad \text{since } 2^5 = 32$$

My notes:

Unit 6: Logarithmic Functions

Unit 6: Logarithmic Functions

Unit 6: Logarithmic Functions

	Skills and Facts
1	• I can rewrite an exponential equation as a log, and a log equation as an exponential.
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Unit 6: Advanced Extensions

	Skills and Facts
1	I can use the change of base formula to rewrite a logarithm with a different base
2	I can identify asymptotes of a logarithmic function

Additional Notes

Unit 6: Logarithmic Functions

Unit 6: Advanced Extensions

	Skills and Facts
1	I can use the change of base formula to rewrite a logarithm with a different base
2	I can identify asymptotes of a logarithmic function
3	

Additional Notes

Unit 6: Logarithmic Functions

Unit 6: Logarithmic Functions

Essential Questions

- How do logarithms relate to exponents?
- How can we solve logarithmic equations? Can we adapt any of our previous understandings to how logarithms work?
- What can we do arithmetically with logarithms?
- What do the graphs of logarithmic functions look like and how do they behave?

Unit 6: Logarithmic Functions

Essential Questions

- How do logarithms relate to exponents?
- How can we solve logarithmic equations? Can we adapt any of our previous understandings to how logarithms work?
- What can we do arithmetically with logarithms?
- What do the graphs of logarithmic functions look like and how do they behave?

LOGARITHMS: DEFINITION OF LOGS

Remember - a logarithm is just like an exponent backwards. Use this understanding and your notes from your “Log Laws Exploration” to complete the following rules

Example: $3^4 = 81$ …so… $\log_3 81 = 4$
We read this as: “log *base* 3 *of* 81 *is* 4.”

A log with no base written has an understood base of _____

Example: $\log 100$ means “log base _____ of ____,” which equals _____

This is called the ____________________________

Fill in the table to move between exponential and logarithmic equations

Exponential Equation	Log Equation
$6^3 = 216$	
$25^{\frac{1}{2}} = 5$	
	$\log_{289} 17 = \frac{1}{2}$
$9^{-2} = \frac{1}{81}$	
	$\log_3 81 = 4$
$u^{-14} = w$	
	$\log_v u = 4$
	$\log_y x = -10$
$12^a = b$	

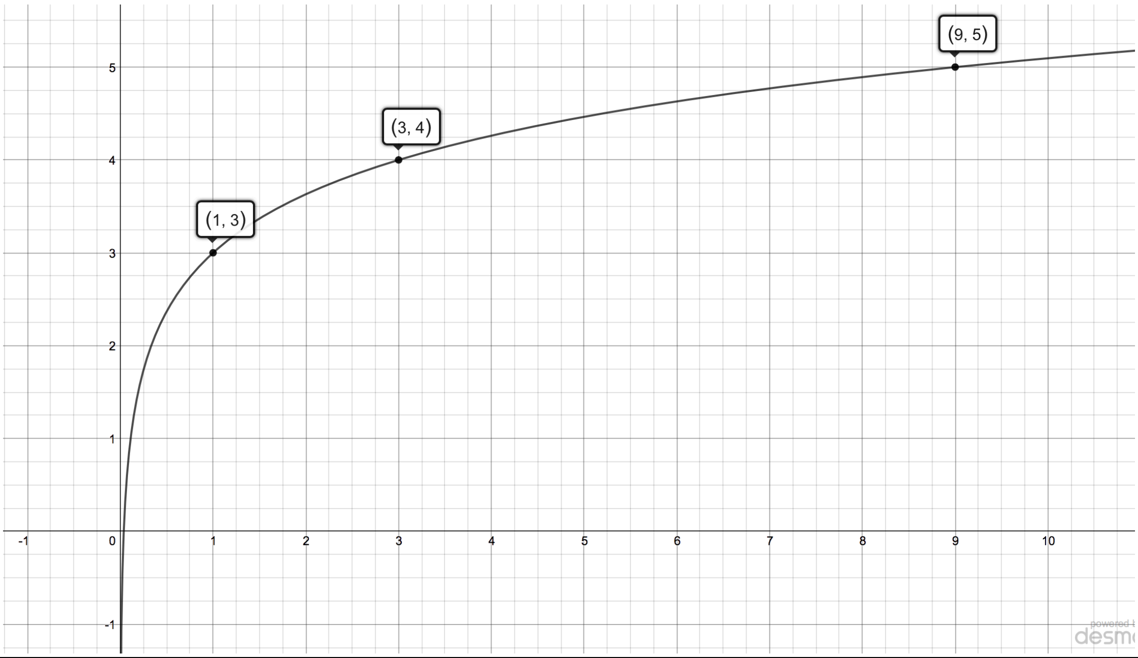
LOGARITHMS: LOG RULES AND EXPONENT RULES	
Remember - a logarithm is just like an exponent backwards. Use this understanding and your notes from your "Log Laws Exploration" to complete the following rules	
EXPONENT RULE	LOG RULE
$x^a * x^b = x^{a+b}$	$\log A + \log B =$
$\dfrac{x^a}{x^b} = x^{a-b}$	$\log A - \log B =$
$(x^a)^b = x^{ab}$	$\log A^n =$
$x^1 = x$	
$x^0 = 1$	
	$\log_b(b^n) = n$
Exceptions – note where logs might not make sense!	
*log*b(*a*) is undefined if *a* is negative.	Why?
*log*b(0) is undefined for any base b.	Why?
$\log_1 1 = n$	This is technically true for any real number, but is not useful.

LOGARITHMS: CHANGE OF BASE FORMULA	
The "Change of Base" formula says that: $$\log_b(x) = \frac{\log_d(x)}{\log_d(b)}$$	
Example: $$\log_3(6) = \frac{\log(6)}{\log(3)}$$	Example: **Convert $\log_3(6)$ to an expression with logs having a base of 5.**
Why does this work?	

LOGARITHMS: GRAPHING

The parent function for logarithmic functions can be written:

Write an equation for the following graph.



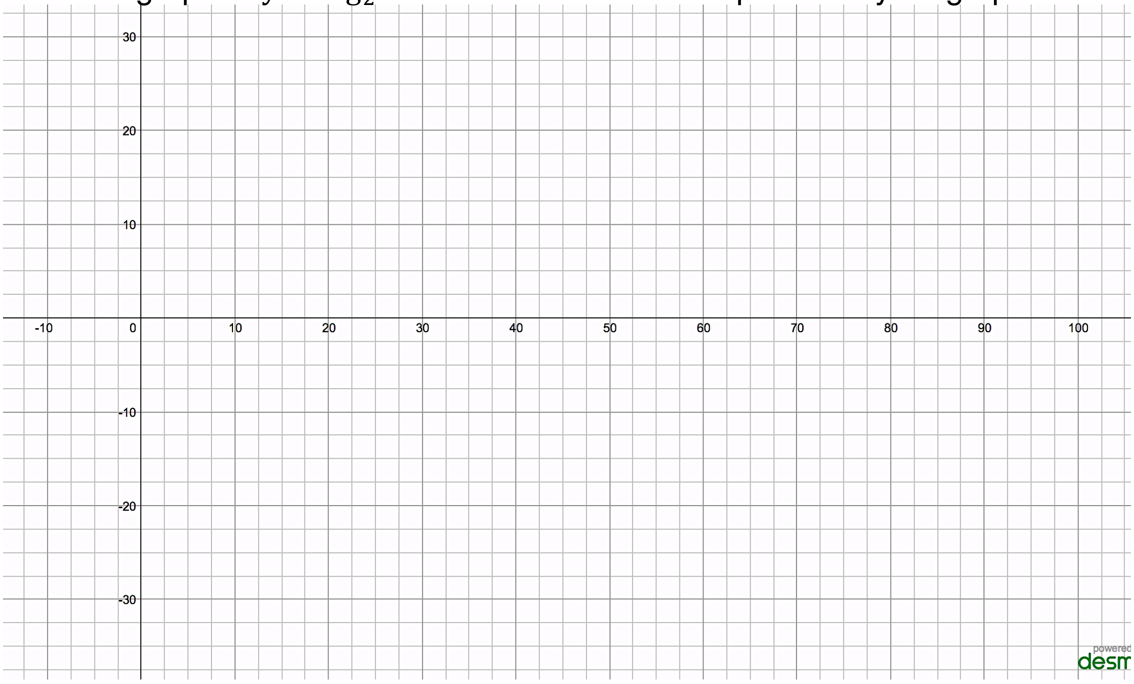
This looks like it is a transformation from the function $\log_{\_\_} x$

- Find the vertical asymptote, and then use the table to help find the equation. Notice that with log functions, it sometimes makes sense to begin with y-values

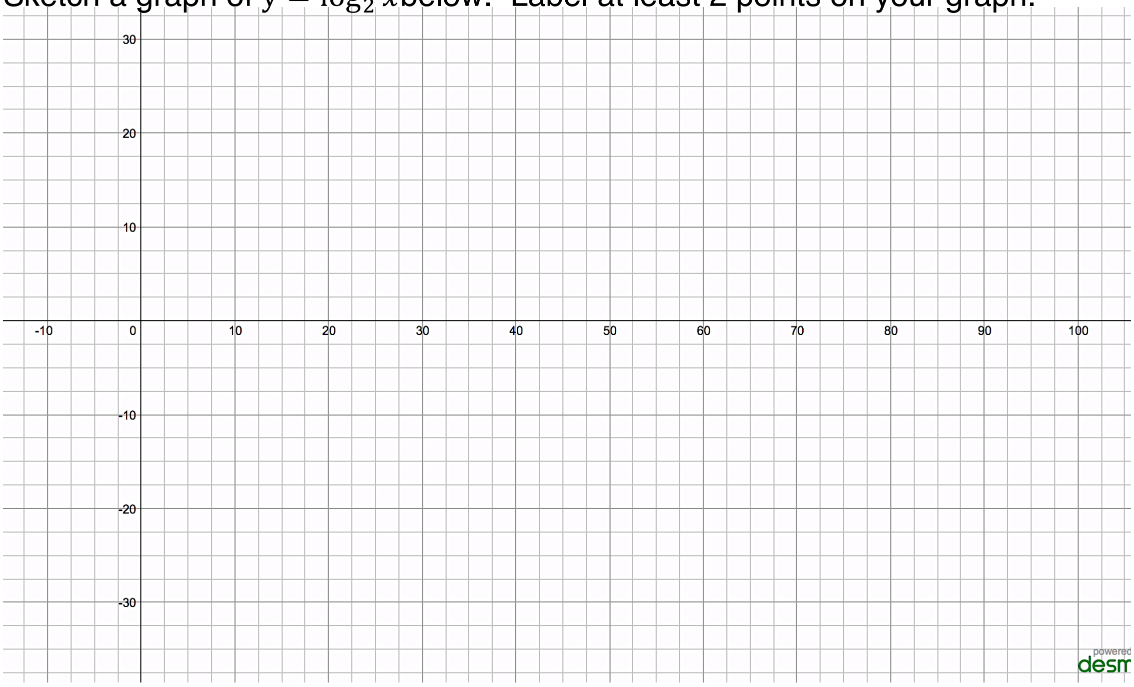
x	y
	0
	1
	2
	3
	4

LOGARITHMS: GRAPHING

Sketch a graph of $y = \log_2 x$ below. Label at least 2 points on your graph.



Sketch a graph of $y = \log_2 x$ below. Label at least 2 points on your graph.



Logs and Exponentials: GRAPHING

Sketch a graph of $y = \log_2 x$ below. Then, sketch a graph of $y = 2^x$ on the same axis.

What is the relationship of the two graphs?

Index Rule	Logarithm Rule
$x^a \times x^b = x^{a+b}$	$\log A + \log B = \log AB$
$\frac{x^a}{x^b} = x^{a-b}$	$\log A - \log B = \log \frac{A}{B}$
$(x^a)^b = x^{ab}$	$\log A^n = n \log A$
$x^1 = x$ & $x^0 = 1$	$\log 10 = 1$ & $\log 1 = 0$

Unit X: Big Ideas

Image source: https://redbooth.com/blog/collaborative-problem-solving

My Notes:

Teacher Notes:

- Working independently is essential to develop personal problem solving skills; working collaboratively can yield deeper and more complex perspectives than working alone. Each is important and related skills can be cultivated and practiced
- Precision in language and computation is essential to arriving at clear and reliable answers
- A problem solver understands what has been done, knows why the process was appropriate, and can support it with reasons and evidence.
- The ability to solve problems is the heart of mathematics.
- The context of a problem determines the reasonableness of a solution.
- There can be different strategies to solve a problem, but some are more effective and efficient than others.
- A problem solver can develop specific strategies for how to understand and begin a task.

Unit X: Problem Solving

Unit X: Problem Solving

Unit X: Problem Solving

Skills and Facts

1 I know that strong problem solvers identify and cultivate specific strategies and habits. I can identify and practice some of these strategies.

2 I know Polya's 4-step problem solving cycle

1. Understand the problem
2. Make a plan
3. Try your plan
4. Check your answers and reflect

3 I can keep trying when I can't find a solution right away

4 I can examine a problem in more than one way

5 I can keep good records of my work

6 I can explain my thinking and tell someone else how I solved a problem

7 I can use precise mathematical notation and language

FINITE DIFFERENCES

First Degree (Linear)

n	$an+b$	
0	b	
		a
1	a+b	
		a
2	2a+b	
		a
3	3a+b	
		a
4	4a+b	
		a
5	5a+b	

Second Degree (Quadratic)

n	an^2+bn+c		
0	c		
		a+b	
1	a+b+c		2a
		3a+b	
2	4a+2b+c		2a
		5a+b	
3	9a+3b+c		2a
		7a+b	
4	16a+4b+c		2a
		9a+b	
5	25a+5b+c		

Third Degree (Cubic)

n	an^3+bn^2+cn+d			
0	d			
		a+b+c		
1	a+b+c+d		6a+2b	
		7a+3b+c		6a
2	8a+4b+2c+d		12a+2b	
		19a+5b+c		6a
3	27a+9b+3c+d		18a+2b	
		37a+7b+c		6a
4	64a+16b+4c+d		24a+2b	
		61a+9b+c		
5	125a+25b+5c+d			

Fourth Degree (Quartic)

n	$an^4+bn^3+cn^2+dn+e$				
0	e				
		a+b+c+d			
1	a+b+c+d+e		14a+6b+2c		
		15a+7b+3c +d		36a+6b	
2	16a+8d+4c+2d+e		50a+12b+2c		24a
		65a+19b+5c+d		60a+6b	
3	81a+27b+9c+3d+e		110a+18b+2c		24a
		175a+37b+7c+d		84a+6b	
4	256a+64b+16c+4d+e		194a+24b+2c		24a
		369a+61b+9c+d		108a+6b	
5	625a+125b+25c+5d+e		302a+30b+2c		
		671a+91b+11c+d			
6	1296a+216b+36c+6d+e				

MATH VOCABULARY (IB Command Terms)	
Definitions of the command terms used in the IB syllabus	
Define:	Give the precise meaning of a word, phrase or physical quantity.
Draw:	Represent by means of pencil lines.
Label:	Add labels to a diagram.
List:	Give a sequence of names or other brief answers with no explanation.
Measure:	Find a value for a quantity.
State:	Give a specific name, value or other brief answer without explanation or calculation.
Annotate:	Add brief notes to a diagram or graph.
Apply:	Use an idea, equation, principle, theory or law in a new situation.
Calculate:	Find a numerical answer showing the relevant stages in the working.
Describe:	Give a detailed account.
Distinguish:	Give the differences between two or more different items.
Estimate:	Find an approximate value for an unknown quantity.
Identify:	Find an answer from a given number of possibilities.
Outline:	Give a brief account or summary.
Analyze:	Interpret data to reach conclusions.
Comment:	Give a judgment based on a given statement or result of a calculation.
Compare:	Give an account of similarities and differences between two (or more) items, referring to both (all) of them throughout.
Construct:	Represent or develop in graphical form.
Deduce:	Reach a conclusion from the information given.
Derive:	Manipulate a mathematical relationship (s) to give a new equation or relationship.
Design:	Produce a plan, simulation or model.
Determine:	Find the only possible answer.
Discuss:	Give an account including, where possible, a range of arguments for and against the relative importance of various factors, or comparisons of alternative hypotheses.
Evaluate:	Assess the implications and limitations.
Explain:	Give a detailed account of causes, reasons or mechanisms.
Predict:	Give an expected result.
Show:	Give the steps in a calculation or derivation.
Sketch:	Represent by means of a graph showing a line and labeled but un-scaled axes but with important features (for example, intercept) clearly indicated
Solve:	Obtain an answer using algebraic and/or numerical methods.
Suggest:	Propose a hypothesis or other possible answer.

Glue this side in your notebook.

The images in this foldable were taken from the following sources:

http://www.cliffsnotes.com/math/algebra/algebra-i/preliminaries-and-basic-operations/square-roots-and-cube-roots

http://www.dominatethegmat.com/2014/01/factors-multiples-and-divisibility-on-the-gmat/

https://www.vismath.eu/en/topics/prime-number

http://www.logan.k12.nj.us/cms/lib02/NJ01000920/Centricity/Domain/284/Fraction-Decimal_Equivalents.jpg

http://calendartemplatesite.info/tag/multiplication-chart-printable-full-page

http://geometryproject.synthasite.com/formulas.php

Mathematician's Tool box from Mathequalslove: http://mathequalslove.blogspot.com/2015/07/free-mathematicians-toolbox-foldable.html

Mathematician's Toolbox

Square Roots

$\sqrt{0}=0$ $\sqrt{16}=4$ $\sqrt{64}=8$

$\sqrt{1}=1$ $\sqrt{25}=5$ $\sqrt{81}=9$

$\sqrt{4}=2$ $\sqrt{36}=6$ $\sqrt{100}=10$

$\sqrt{9}=3$ $\sqrt{49}=7$

Cube Roots

$\sqrt[3]{0}=0$ $\sqrt[3]{64}=4$ $\sqrt[3]{512}=8$

$\sqrt[3]{1}=1$ $\sqrt[3]{125}=5$ $\sqrt[3]{729}=9$

$\sqrt[3]{8}=2$ $\sqrt[3]{216}=6$ $\sqrt[3]{1000}=10$

$\sqrt[3]{27}=3$ $\sqrt[3]{343}=7$

Prime Number Chart

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

Multiplication Chart

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225

Divisibility Rules

A number is divisible by. . .		Divisible	Not Divisible
2	if the last digit is even (0, 2, 4, 6, or 8).	3,978	4,975
3	if the sum of the digits is divisible by 3.	315	139
4	if the last two digits form a number divisible by 4.	8,512	7,518
5	if the last digit is 0 or 5.	14,975	10,978
6	if the number is divisible by both 2 and 3	48	20
9	if the sum of the digits is divisible by 9.	711	93
10	if the last digit is 0.	15,990	10,536

Fraction/Decimal Equivalents

$\frac{1}{2}$	0.5	$\frac{1}{8}$	0.125	$\frac{1}{11}$	$0.\overline{09}$	$\frac{1}{16}$	0.0625
$\frac{1}{3}$	$0.\overline{3}$	$\frac{2}{8}$	0.25	$\frac{2}{11}$	$0.\overline{18}$	$\frac{2}{16}$	0.125
$\frac{2}{3}$	$0.\overline{6}$	$\frac{3}{8}$	0.375	$\frac{3}{11}$	$0.\overline{27}$	$\frac{3}{16}$	0.1875
$\frac{1}{4}$	0.25	$\frac{4}{8}$	0.5	$\frac{4}{11}$	$0.\overline{36}$	$\frac{4}{16}$	0.25
$\frac{2}{4}$	0.5	$\frac{5}{8}$	0.625	$\frac{5}{11}$	$0.\overline{45}$	$\frac{5}{16}$	0.3125
$\frac{3}{4}$	0.75	$\frac{6}{8}$	0.75	$\frac{6}{11}$	$0.\overline{54}$	$\frac{6}{16}$	0.375
$\frac{1}{5}$	0.2	$\frac{7}{8}$	0.875	$\frac{7}{11}$	$0.\overline{63}$	$\frac{7}{16}$	0.4375
$\frac{2}{5}$	0.4	$\frac{1}{9}$	$0.\overline{1}$	$\frac{8}{11}$	$0.\overline{72}$	$\frac{8}{16}$	0.5
$\frac{3}{5}$	0.6	$\frac{2}{9}$	$0.\overline{2}$	$\frac{9}{11}$	$0.\overline{81}$	$\frac{9}{16}$	0.5625
$\frac{4}{5}$	0.8	$\frac{3}{9}$	$0.\overline{3}$	$\frac{10}{11}$	$0.\overline{90}$	$\frac{10}{16}$	0.625
$\frac{1}{6}$	$0.1\overline{6}$	$\frac{4}{9}$	$0.\overline{4}$	$\frac{1}{12}$	$0.08\overline{3}$	$\frac{11}{16}$	0.6875
$\frac{2}{6}$	$0.\overline{3}$	$\frac{5}{9}$	$0.\overline{5}$	$\frac{2}{12}$	$0.1\overline{6}$	$\frac{12}{16}$	0.75
$\frac{3}{6}$	0.5	$\frac{6}{9}$	$0.\overline{6}$	$\frac{3}{12}$	0.25	$\frac{13}{16}$	0.8125
$\frac{4}{6}$	$0.\overline{6}$	$\frac{7}{9}$	$0.\overline{7}$	$\frac{4}{12}$	$0.\overline{3}$	$\frac{14}{16}$	0.875
$\frac{5}{6}$	$0.8\overline{3}$	$\frac{8}{9}$	$0.\overline{8}$	$\frac{5}{12}$	$0.41\overline{6}$	$\frac{15}{16}$	0.9375
$\frac{1}{7}$	$0.\overline{142857}$	$\frac{1}{10}$	0.1	$\frac{6}{12}$	0.5		
$\frac{2}{7}$	$0.\overline{285714}$	$\frac{2}{10}$	0.2	$\frac{7}{12}$	$0.58\overline{3}$		
$\frac{3}{7}$	$0.\overline{428571}$	$\frac{3}{10}$	0.3	$\frac{8}{12}$	$0.\overline{6}$		
$\frac{4}{7}$	$0.\overline{571428}$	$\frac{4}{10}$	0.4	$\frac{9}{12}$	0.75		
$\frac{5}{7}$	$0.\overline{714285}$	$\frac{5}{10}$	0.5	$\frac{10}{12}$	$0.8\overline{3}$		
$\frac{6}{7}$	$0.\overline{857142}$	$\frac{6}{10}$	0.6	$\frac{11}{12}$	$0.91\overline{6}$		
		$\frac{7}{10}$	0.7				
		$\frac{8}{10}$	0.8				
		$\frac{9}{10}$	0.9				

LENGTH

Metric	Customary
1 kilometer = 1000 meters	1 mile = 1760 yards
1 meter = 100 centimeters	1 mile = 5280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

CAPACITY AND VOLUME

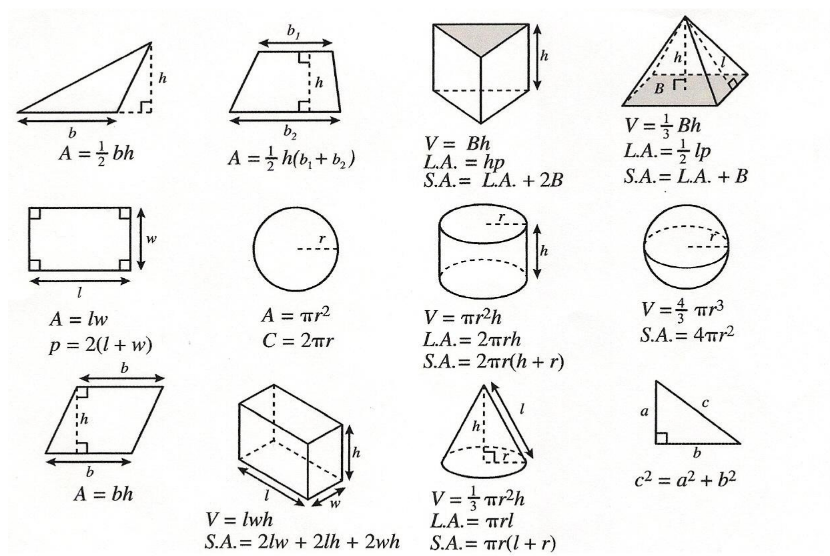
Metric	Customary
1 liter = 1000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 ounces

MASS AND WEIGHT

Metric	Customary
1 kilogram = 1000 grams	1 ton = 2000 pounds
1 gram = 1000 milligrams	1 pound = 16 ounces

TIME

1 year = 365 days
1 year = 12 months
1 year = 52 weeks
1 week = 7 days
1 day = 24 hours
1 hour = 60 minutes
1 minute = 60 seconds



Trigonometry Table

A	SIN(A)	COS(A)	Tan(A)
0	0.0000	1.0000	0.0000
1	0.0175	0.9998	0.0175
2	0.0349	0.9994	0.0349
3	0.0523	0.9986	0.0524
4	0.0698	0.9976	0.0699
5	0.0872	0.9962	0.0875
6	0.1045	0.9945	0.1051
7	0.1219	0.9925	0.1228
8	0.1392	0.9903	0.1405
9	0.1564	0.9877	0.1584
10	0.1736	0.9848	0.1763
11	0.1908	0.9816	0.1944
12	0.2079	0.9781	0.2126
13	0.2250	0.9744	0.2309
14	0.2419	0.9703	0.2493
15	0.2588	0.9659	0.2679
16	0.2756	0.9613	0.2867
17	0.2924	0.9563	0.3057
18	0.3090	0.9511	0.3249
19	0.3256	0.9455	0.3443
20	0.3420	0.9397	0.3640
21	0.3584	0.9336	0.3839
22	0.3746	0.9272	0.4040
23	0.3907	0.9205	0.4245
24	0.4067	0.9135	0.4452
25	0.4226	0.9063	0.4663
26	0.4384	0.8988	0.4877
27	0.4540	0.8910	0.5095
28	0.4695	0.8829	0.5317
29	0.4848	0.8746	0.5543
30	0.5000	0.8660	0.5774
31	0.5150	0.8572	0.6009
32	0.5299	0.8480	0.6249
33	0.5446	0.8387	0.6494
34	0.5592	0.8290	0.6745
35	0.5736	0.8192	0.7002
36	0.5878	0.8090	0.7265
37	0.6018	0.7986	0.7536
38	0.6157	0.7880	0.7813
39	0.6293	0.7771	0.8098
40	0.6428	0.7660	0.8391
41	0.6561	0.7547	0.8693
42	0.6691	0.7431	0.9004
43	0.6820	0.7314	0.9325
44	0.6947	0.7193	0.9657
45	0.7071	0.7071	1.0000

A	SIN(A)	COS(A)	Tan(A)
45	0.7071	0.7071	1.0000
46	0.7193	0.6947	1.0355
47	0.7314	0.6820	1.0724
48	0.7431	0.6691	1.1106
49	0.7547	0.6561	1.1504
50	0.7660	0.6428	1.1918
51	0.7771	0.6293	1.2349
52	0.7880	0.6157	1.2799
53	0.7986	0.6018	1.3270
54	0.8090	0.5878	1.3764
55	0.8192	0.5736	1.4281
56	0.8290	0.5592	1.4826
57	0.8387	0.5446	1.5399
58	0.8480	0.5299	1.6003
59	0.8572	0.5150	1.6643
60	0.8660	0.5000	1.7321
61	0.8746	0.4848	1.8040
62	0.8829	0.4695	1.8807
63	0.8910	0.4540	1.9626
64	0.8988	0.4384	2.0503
65	0.9063	0.4226	2.1445
66	0.9135	0.4067	2.2460
67	0.9205	0.3907	2.3559
68	0.9272	0.3746	2.4751
69	0.9336	0.3584	2.6051
70	0.9397	0.3420	2.7475
71	0.9455	0.3256	2.9042
72	0.9511	0.3090	3.0777
73	0.9563	0.2924	3.2709
74	0.9613	0.2756	3.4874
75	0.9659	0.2588	3.7321
76	0.9703	0.2419	4.0108
77	0.9744	0.2250	4.3315
78	0.9781	0.2079	4.7046
79	0.9816	0.1908	5.1446
80	0.9848	0.1736	5.6713
81	0.9877	0.1564	6.3138
82	0.9903	0.1392	7.1154
83	0.9925	0.1219	8.1443
84	0.9945	0.1045	9.5144
85	0.9962	0.0872	11.4301
86	0.9976	0.0698	14.3007
87	0.9986	0.0523	19.0811
88	0.9994	0.0349	28.6363
89	0.9998	0.0175	57.2900
90	1.0000	0.0000	∞

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Problem Solving Protocol: Grade10

The following cycle will help you when you come across a situation or a problem that you have not seen before. The FOUR KEY STEPS are essential, but the specifics listed below each big step are just suggestions. You do not need to do every one of these steps. Just pick and choose the ones that work for each situation – or invent your own!

1. Understand the Problem

- Read and **decode**
- Identify and highlight important information
- Decide what information is not useful
- Try to rewrite the problem in your own words
- Rewrite the question in your own words for clarity
- Rewrite the problem with mathematical notation
- Find a definition for words that you don't know
- Make a table comparing the information you know, and what you don't know
- Make a picture or diagram to help you understand the problem
- Check to see if you have enough information to solve the problem

2. Make a Plan

- Make an orderly list
- Make a table
- Draw a picture
- Look for patterns
- List all of the things you know
- Create and solve a simpler version of your problem - consider a special case
- Eliminate impossible or absurd answers
- Use a variable to represent an unknown
- Work backwards
- Use a formula
- Make a model
- Be fearless - willing to take risks!
- Make a connection to something you have learned before
- Identify resources that you can use
- **Estimate** an answer

3. Try Your Strategy

- Guess and check
- Extend your table or your list
- Refine and analyze your picture
- Attend to precision – check each step as you work
- Make a convincing argument
- Persist with the plan you have chosen
- Discard your plan and choose another (this is how mathematics is done, even by professionals!)
- **Show your work** – keep good records of what you've done

4. Check Your Answers and Reflect

- Does your answer make sense when you put it back in the context of the original question? Does it work? How does your answer compare to your original estimate?
- Are you confident that you have come to a correct solution?
- Was your strategy efficient?
- Can you think of a different way to solve the problem?
- Was there something that you realized along the way?
- Can you use your method to solve other problems? Can you make a generalization?
- Did you spot any patterns?

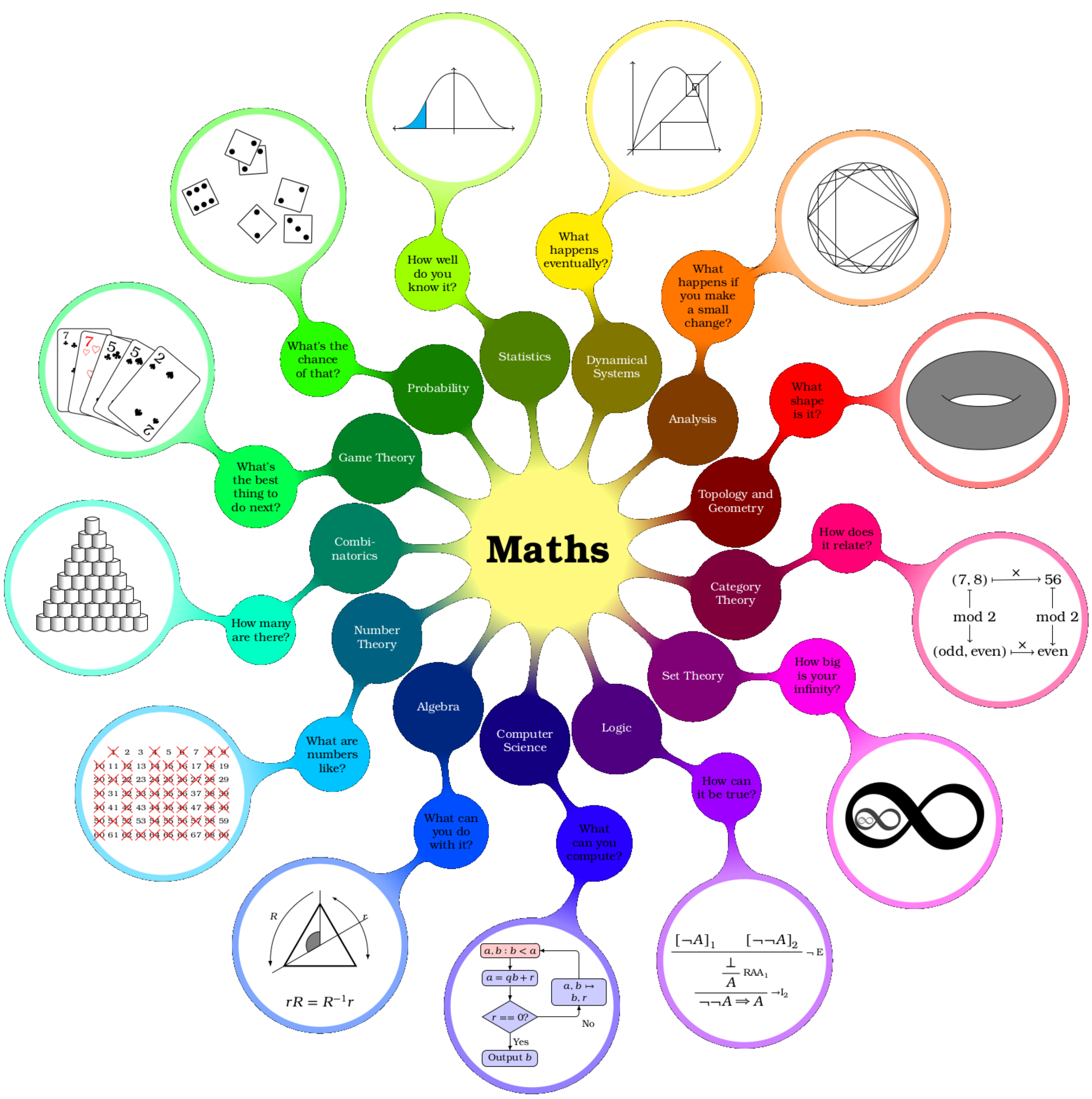
Problem Solving Rubric: Grade 10

This work is intended to support students in becoming **confident** and **independent** problem solvers who are comfortable making attempts and taking risks when presented with novel situations.

	Distinguished	Proficient	Learning
Understanding (10%)	• States the problem clearly and identifies important information and underlying issues; clearly defines the problem and outlines necessary objectives	• Adequately defines the problem and identifies important information	• Needs assistance to identify important information or get started; problem is defined incorrectly or too narrowly. Key information is missing or incorrect.
Strategic Planning (30%)	• Evidence of careful analysis • Evidence of a clear and concise plan to solve the problem, with alternative strategies	• Evidence of analysis • Evidence of an adequate plan to solve the problem	• Little evidence of a coherent plan to solve the problem, or evidence of a plan that is not adequate
Implementation (30%)	• Provides a logical interpretation of the findings and clearly solves the problem, offering alternative solutions • Uses subject-area strategies, tools, and knowledge • Applies procedures and follows the plan to conclusion. • Attends to precision and records all work to allow for backtracking	• Provides an adequate interpretation of the findings and solves the problem • Uses subject-area strategies, tools, and knowledge • Applies procedures and follows the plan to conclusion. • Attends to precision	• Applies inappropriate procedures or only partially applies correct procedures • Needs reminders to use subject-area strategies, tools, or knowledge to solve problems. • Does not interpret the findings/reach a conclusion.
Communication (10%)	• Clearly and concisely articulates the problem-solving process and describes how it was applied to the current problem • Uses a representation that is exceptional in its mathematical precision • Comprehensive record of process and data. Includes detailed information to allow repetition based only on written notes. • Explains why certain information is essential to the solution	• Describes the problem-solving process • Uses a representation that clearly depicts the problem • Process and data are summarized and organized, but may lack some details or some explanation necessary for repetition • Explains why procedures are appropriate for the problem	• Requires assistance to describe the problem solving processes • Uses a representation that gives some but not all important information about the problem • Notes aren't organized and results cannot be easily found. Experiments or other work cannot be repeated because of lack of information. • Needs assistance to assess why procedures or techniques were applied to the problem
Answer (10%)	• Correct solution of problem and made a general rule about the solution or extended the solution to a more complex situation **or** partial credit for solution that is incorrect but has a strong justification	• Correct solution with justification **or** partial credit for solution that is incorrect but has a strong justification	• Copying or computational error, partial answer, no answer statement, answer labeled incorrectly, perhaps no answer or wrong answer based on an inappropriate plan
Reflection (10%)	• Evidence of reflection on problem solving processes, and evaluation of how well they worked; willingness to make changes when necessary • Critical reflection on problem-solving techniques, strategies, and results. Identifies those most helpful to self. Offers clear insights regarding self-knowledge	• Evidence of reflection on problem solving processes by thinking about what I did well and what I can do better. • Can identify problem-solving techniques that are most helpful, but may not be able to clearly summarize self-knowledge.	• Difficulty revealing insights about own learning. • Difficulty discussing relevance of problem-solving techniques.

Sweet math poster taken from http://loopspace.mathforge.org/CountingOnMyFingers/PiecesOfMath/#section.1

I removed the black background for easier printing.



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