FORMAT GUIDE

Clarifying notes about the unit

Concept generalizations



Algebra 2 Concept-Based Map DRAFT

Algebra 2 is a language that can express relationships and reveal patterns between two related variables. It gives us the tools to apply and relate functions to equations and to create mathematical models, which we can use to make inferences, predictions and to draw conclusions.

The course starts by providing a general overview of different families of functions so students can identify key features that distinguish families, and recognize that some families work well to model particular situations.

Subsequent units give students opportunities to deepen knowledge of each function family including transformations, inverses, understanding the effects of restricting domain and range, solving for key features including maxima and minima, roots, intercepts, symmetry, and end behavior and how to apply these tools to real life and abstract situations.

This course culminates with applications of functions. Students analyze and interpret statistics and use probability to make predictions and inferences from data.

In conjunction with the above, this course is intended to deepen students' relationships with the mathematical practices and help them to become confident and independent problem-solvers.







Algebra 2 Timeline



This unit is intended to set the culture and expectations of class for the school year.

Students will understand the importance of listening to how others make sense of their work to arrive at common understandings, and will contextualize themselves as valuable and valued parts of a thinking mathematical community. They will understand that mathematics is a study of patterns and relationships.



This unit is primarily about the descriptive, not the quantitative. Students will make connections to and build on their prior understandings of functions and develop a deeper understanding of how two quantities relate to each other.

Students will understand that functions articulate how two quantities are related. They will understand how functions are notated and described, and ways in which we compare functions. Students will understand how and why we represent different functions and function families in different ways - graphically, algebraically, through verbal descriptions, and through tables.

Mathematical Models Students will understand that graphs can describe situations **Function Families** from the world • Functions are mappings of inputs to exactly one output Students will create graphs to model situations • Families of functions have distinct shapes and characteristics Modeling often requires moving among different representations. Identify and articulate differences between function families Models are imperfect but often useful. • Key features can help distinguish function families. Analyze key features of graphs. Students will understand that mathematical models can illustrate and reveal aspects of real situations, and will create Students will understand that function families share similar and analyze graphs to demonstrate this understanding. graphs, behaviors, and properties **Unit I:** Families of Functions Parent Functions Linear **Transformations** Exponential · Students will sort functions into families, and Representation Logarithmic will identify their distinct characteristics Polynomial • Students will connect Graphic, Algebraic, Students will relate transformed functions to Square Root Tabular, and Verbal Representations parent functions Absolute Value · Students will interpret graphs and use them • Function notation f(x), g(x), etc. • Trig/Periodic to describe situations Quadratic Students will interpret equations and use Students will understand that functions within them to describe situations

Students will understand that functions within a family share specific characteristics.

a family are transformations of the parent function. Students will practice with the precise grammar and vocabulary in math by using function notation.

Students will understand that functions can be represented in multiple, equivalent ways

In Unit 1 much of the work is descriptive and general. From Unit 2 forward, students are asked to calculate, quantify and formalize their analysis.

Students will understand that constant rate of change is the key feature that defines linear functions, and that given one location and a rate of change, they can predict any other location. Students will understand how graphs, equations, verbal descriptions, and tables can describe different aspects linear functions. Students understand that absolute value functions, linear systems, and linear inequalities are related to and extensions of linear functions.

Linear Equations

- Slope
- Slope-Intercept form
- Standard form
- Point-slope Form
- A solution is a value that makes a function rule true.
- Writing equations

Students will write equations to represent situations. They will manipulate equations from one form to another, and understand that different but equivalent forms reveal different aspects of a function.

Representation

- Graphic, Algebraic, Verbal, Numeric
- Move with facility between G>A>V>N

Students will understand that functions can be represented in multiple, equivalent ways, and that each way can be helpful for different situations, and can reveal and illuminate different characteristics.

Absolute Value

- Graphs and transformations
- Writing and solving equations/inequalities
- Vertex

Students will recognize the characteristic shape of an absolute value graph, and will recognize transformations from the absolute value parent function y=|x|.

Function Rules

- nth term
- linear/arithmetic sequence
- Input/Output
- Continuous/Discreet
- Arithmetic Sequences and Series including summation
- Sigma Notation
- Common Differences

Students will understand that function rules describe the quantitative relationships between variables.

Constant Rate of Change

- Linear functions are characterized by a constant rate of change.
- A function's rate of change and initial value determine its other properties and behaviors.
- y-intercept and transformations
- Use a function's rate of change to predict future states/generate next steps

Students will know that linear functions are characterized by a constant rate of change. They will understand that the initial value and rate of change can be used to make predictions.

Unit 2: Linear Functions

Linear Inequalities

- Graphing
- Notation
- Linear Programming

Students will understand that inequalities in 2 variables divide the plane into two regions, and will create and analyze graphs of 2 variable inequalities.

Linear Systems

• Elimination, Substitution, Graphing

Students will recognize that systems of equations (or Inequalities) contain functions that share the same set of variables. They will understand that a solution simultaneously makes each function rule in a system of equations (or inequalities) true. They will understand that the solution to a system of equations (or inequalities) can be represented in multiple, equivalent ways. Students investigate exponential growth and decay as key features that define exponential functions, and write equations that describe the relationship between quantities for exponential functions including geometric sequences.

Students recognize and identify key features of exponential growth and decay. They understand how to manipulate exponential equations by using the laws of exponents and roots. Students understand how to communicate using precise mathematical language and forms, and how to compare exponential to linear models. Students understand how to differentiate between geometric and arithmetic sequences.



This unit is intended as an overview of right angle trigonometry and vector addition. The work in this unit will scaffold students so they gain the skills they will need for the work they will be doing in Grade 10 Physics.

Students will understand how to find all measurements in right triangles given either one side and one angle or two side lengths, and will recognize relationships between circles and triangles. Students will understand that vector addition can be modeled with right-angled triangles, and they will understand how to create graphs and models using the sine function.



or two side lengths. Students will understand that equations for circles can be formed by using the Pythagorean Theorem and the relationships in right-angled triangles, and that Trig functions can be derived in this way.

Students will understand key features that define quadratic functions, and how to write equations that describe the relationship between quantities for quadratics. Students will understand how to identify key features of quadratic models through analysis of graphs. They will understand that they can solve quadratic equations through multiple methods and will identify the advantages of each method. They will understand how to manipulate the graphs of quadratic equations by transforming the parent function $y=x^2$ and will understand that they can transfer this understanding to analyze characteristics of higher degree polynomials.

Representation **Quadratic Equations** • Vertex form · Multiple ways to solve quadratic equations Standard form Complete the Square Factored form Quadratic Formula • Graphing Factoring • Tables > common 2nd differences • 1,2, or no real roots • Powers of *i*, imaginary and complex solutions Students will understand that equivalent representations of a function highlight different properties. They will demonstrate facility in using the Students will understand that there are many ways to solve tools of algebra to move between different forms. a quadratic equation, and that there will be one, two, or no real solutions. Students will understand that if there are no real solutions, there are solutions, using *i* on the set of complex numbers. **Transformations** Vertical and Horizontal shifts Vertical and Horizontal stretches/flips Symmetry of quadratics Unit 5: Quadratic Functions Students will understand that all graphs of functions within a family (including quadratics) are transformations of the parent function. **Characteristics Polynomials** • Quadratic equations are distinguished by x^2 , and will relate Local Minima and Maxima this concept to the area of a quadrilateral. End Behavior Characteristic shape of a Parabola Graphing Roots (Solutions), Vertex, Factoring Line of Symmetry, Minimum/Maximum • Quadratic and Cubic sequences and common differences - nth term Graphs rules

Students will understand that quadratic functions are distinguished by x^2 , and that quadratic graphs make a distinct shape called a parabola

Students will understand that concepts of quadratics, and algebraic techniques can be used to help understand high degree polynomials.

Students understand key features that define Logarithmic functions, the inverse relationship between logs and exponentials, and the implications of that relationship to the rate of change of logs.



Students apply their knowledge of functions to analyze and interpret statistical information and to make predictions and inferences from data.

Students understand how to choose appropriate visual models to illustrate connections between variables and how to use statistics to support and critique arguments and make predictions



Problem solving is integrated through the entire course, but it is important to place emphasis on supporting students through this process. The materials and ideas connected to this unit are intended to support students in becoming confident and independent problem-solvers. Some of the learning experiences are connected to specific course content, and some are targeted to discreet problem-solving skills. This work should be included wherever possible throughout the course.



Unit 0: Intro to Algebra 2

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Essential Understandings	Compelling Questions	
A learning community has to be a safe environment for risk taking and experimentation	What is important in a learning community?	
A growth mindset, and willingness to take risks and make mistakes are essential to learning and doing mathematic? Successful mathematicians (and math students!) identify, practice, and cultivate specific habits	 What makes a strong math thinker? What makes a strong math student? 	I. Growth Mindset
Mathematics consists of thinking in addition to procedures, and there are specific steps and systems that we can apply as problem solvers	 What makes a strong problem solver, and how can we learn to be strong problem solvers? 	 Math Practices Habits of Mind Pattern
• Social learning and collaboration are important to gaining understanding, especially in math. Working independently is a way to make sure that you have learned and can apply concepts.	 How can collaboration help us to learn (math)? When and why is it important to work independently? 	 Relationship Precise Persevere
Precise work and language are important in communicating and sharing ideas in math.	 What are the different ways of communicating mathematics with clarity? 	9. Reason
 Mathematics is a study of patterns and relationships. These patterns provide insights into potential relationships. 	How can patterns be used to make predictions?	
 Patterns and relationships can be represented numerically, graphically, symbolically, and verbally. Each representation can help us to understand these relationships in different ways. 	 What are the different ways that we can illustrate relationships, and how do we choose between them? 	
Critical Content	Key Skills	
 Mathematics as the study of patterns Understanding of growth and fixed mindsets and the impact that each has on learning 	Collaborative workIndependent workPerseverance	
Understanding that every member contributes to the culture of a learning community	Arguing and CritiquingFinding patterns	
There are specific strategies and habits for problem-solving that can be cultivated and learned	Finding and using structurePrecision in arithmetic and language	

Unit I: Families of Functions

 Essential Understandings A function is a correspondence between two sets, X and Y, in which each element of X is matched to one and only one element of Y. The set X is called the domain of the function. Compelling Questions What is a function? What is a function? I. Linear Exponent Ourdentic 	ulary al
 A function is a correspondence between two sets, <i>X</i> and <i>Y</i>, in which each element of <i>X</i> is matched to one and only one element of <i>Y</i>. The set <i>X</i> is called the domain of the function. What is a function? I. Linear 2. Exponent 3. Our dratic 	al
 Function families share similar graphs, behaviors, and properties; functions within a family are transformations of the parent function What features do functions within the same family share? What makes a particular function family unique from other types of functions? What features do functions within the same family share? What makes a particular function family unique from other types of functions? 	ic etric
 Functions can be represented in multiple, equivalent ways. Each representation has its own advantages What are the different ways that we can illustrate functions, and how do we choose between them? 7. Absolute 10, 20, 20, 20, 20, 20, 20, 20, 20, 20, 2	/alue tion
 Mathematical models can illustrate and reveal aspects of real situations; graphing assists in our analysis and understanding. Why do we create mathematical models and what do the key features of the graph of a function reveal? Why do we create mathematical models and what do the key features of the graph of a function reveal? 	ent/
 The grammar and vocabulary of math, including function notation, allow us to communicate precisely. We can make explicit use of this precision to make strong arguments Why are there different methods of notation for functions? Why is it important to use precise notation? Why are there different methods of notation for functions? Why is it important to use precise notation? Interval of the precise of t	Dependent variable 12. Domain 13. Range
Critical Content Key Skills	
 Analyze key features of graphs, including: Increasing interval, decreasing interval, intercepts, periodicity, minimum/maximum, domain and range, end behavior Analyze key features using appropriate vocabulary Identify the general shape and behavior of different function families 	
 Recognize the differences between average rate of change vs. Constant Rate of Change Use minimums, maximums and intercepts to describe a function and represent it in multiple ways 	
Relate graphs and tables to situations; tell the "story" of a graph Determine a function's family using its key points, and shape	
 Creation of scatterplots, Time-Distance Graphs Interpret a situation, graph, table of values, or equation using the 	
 Distinguish between function families Recognize transformations from a parent function Identify an appropriate function family for a situation and defend 	
 Make connections between multiple representations Make connections between multiple representations 	
 Understanding that mathematical models can illustrate and reveal aspects of real situations Justify and compare representations of different function families and describe similarities and differences. Given one representation, create another. 	
Compare functions within a family and describe the transformations from the parent function	
Create and analyze graphs to illustrate and reveal aspects of real situations	

Unit 2: Linear Functions

Essential Understandings

- Linear equations represent real and abstract situations characterized by a constant rate of change. Their graphs always make straight lines
- Equations can be manipulated from one form to another, and different but equivalent forms reveal different aspects of a function
- Function rules describe the quantitative relationships between variables. Tables are useful for beginning a pattern, and nth term expressions are useful for making more distant predictions
- The initial value and rate of change of a linear function can be used to make predictions.
- An absolute value graph has a characteristic "V" shape, and transformations from the absolute value parent function y=|x| share this characteristic. Absolute value represents distance from zero.
- Systems of equations represent situations with more than one constraint, and contain functions that share the same set of variables. A solution simultaneously makes each function rule in a system of equations true, and the solution to a system of equations can be represented in multiple, equivalent ways.
- Linear inequalities in 2 variables divide the plane into two regions. They are related to linear equations, but can be used to model situations with a maximum or minimum

Critical Content

- · Linear functions are characterized by a constant rate of change
- Recognize and solve linear combinations
- Solving linear equations
- Fitting lines to data on a graph (how to find a line of best fit), through sketching and through technology.
- Constant rate of change problems
- Solving absolute value problems
- Geometrically, the absolute value of a number is its distance on a number line from 0; Algebraically, the absolute value of a number equals the nonnegative square root of its square.

Compelling Questions

- What types of relationships can be modeled by linear functions, and what do math models of these relationships look like?
- Why are there different forms for notating equations of lines, and how can we decide in which format to write a linear equation?
- How can we write recursive or explicit rules for linear situations, and why do we write function rules?
- · How can we use linear equations to make predictions?
- What are the key features of absolute value relationships and graphs?
- What can we do with a system of equations/inequalities that we cannot do with a single equation/inequality? How do we find solutions to systems?
- How are linear inequalities similar or different from linear equations, and when are they useful?

Key Skills

- Determine the slope and intercepts of a line given its equation.
- Find an equation of a line given two points on it or given a point on it and its slope.
- Recognize properties of linear functions.
- Model constant-increase or constant-decrease situations, linear combination situations, mixture problems, and situations leading to piecewise linear functions or to linear step functions.
- Create and analyze graphs of linear functions, piecewise linear functions, step functions, and sequences.
- Solve absolute value equations and inequalities algebraically and graphically.
- Write and analyze equations and inequalities that describe the relationship between quantities for linear functions.
- Write nth term expressions.

Vocabulary

- I. Linear Equation
- 2. nth term
- 3. Arithmetic Sequence
- 4. Arithmetic Series
- 5. Slope
- 6. x- and y-intercepts
- 7. Parallel
- 8. Perpendicular
- 9. Common Difference
- 10. System of Equations
- II. Inequality

Unit 3: Exponential Functions

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 Essential Understandings There are rules and procedures for arithmetic with exponential expressions which can save us from doing repeated or redundant calculations Geometric sequences relate to and behave like exponential functions, and can be used to model real situations. Geometric and Arithmetic sequences have specific similarities and differences. Exponential growth and decay have characteristic shape and behavior, and are substantially different from linear growth. Exponential graphs have a characteristic shape, and all exponential graphs are transformations from the parent function y=b^x. Roots can be expressed as rational exponents. Changing the form of expressions and equations is a way to communicate with specific mathematical grammar. Equations can be manipulated from one form to another, and different but equivalent forms reveal different aspects of a function 	 Compelling Questions Why is it helpful to learn about the mechanics of exponents, and how do exponents work? How are geometric sequences related to exponential functions, and where do they show up in the world? How are they similar or different from arithmetic sequences? What does exponential growth or decay look like, and how does it compare to linear growth? How can graphs help us to visualize these differences? How are expressions involving radicals and exponents related? Why are there different forms for notating expressions and equations, and how can we decide in which format to write an exponential equation? 	 Vocabulary 1. Exponent 2. Exponential Growth 3. Exponential Decay 4. Base 5. Rational Exponent 6. Square Root 7. Power 8. nth root 9. Geometric sequence 10. Compound interest
 Critical Content Product of powers postulate Quotient of powers theorem Negative exponent theorem Rational exponent theorem Manipulate radical expressions Understand graphs of exponential equations Calculate compound interest nth term expressions for geometric sequences 	 Key Skills Manipulate exponential equations by using the laws of exponents and roots Write a radical function or expression as an equivalent power function or expression. Represent exponential functions numerically, algebraically, and graphically. Solve radical equations graphically and algebraically, and check for extraneous roots. Write equivalent expressions involving radicals and exponents, including negative exponents. Evaluate expressions involving radicals and exponents. Sketch graphs of exponential functions and create graphs using technology 	

Unit 4: Trigonometric Functions

Essential Understandings	Compelling Questions	
 A scalar quantity has magnitude, and a vector quantity has both direction and magnitude. 	What is the difference between a scalar and a vector?	Vocabulary
 Vector addition can be used to solve problems from the world involving force and direction. There are several ways to solve these problems using trig or graphing. 	 How and why do we use vectors? 	 Ratio Sine Cosine
 We can find all measurements in right triangles given either one side and one angle, or two side lengths. 	 If we know the lengths & measures of SOME sides & angles of a triangle, when & how can we find all the others? 	 Figonometry Tangent Vector
 Equations for circles can be formed by using the Pythagorean Theorem and the relationships in right-angled triangles, and that Trig functions can be derived in this way. 	 How are equations for circles related to right triangles? 	7. Magnitude 8. Direction 9 Scale
 The graph of a sine functions has a characteristic shape and behavior, which is an excellent model for certain situations. (eg. hours of daylight over time, height of tides, etc.) 	 What do the graphs of Trig functions look like and how do they behave? 	10. Modulation11. Period12. Amplitude
Critical Content	Key Skills	
 Model forces in the world with vectors, and solve problems using this model 	 Recognize a situation that modulates and can be represented well with a sine function. 	
 Vector addition Craphing size functions, and analysis of these graphs 	 Create and transform a sine function to match a situation. (eg. height of a ferris wheel over time.) 	
 Graphing sine functions, and analysis of these graphs. Solving for missing sides/angles of triangles. 	Use right angle trig to solve vector problems	
	Use graphing method (protractor and ruler) to solve vector problems	

Unit 5: Quadratic Functions

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Essential Understandings	Compelling Questions	
 Quadratic functions are distinguished by x²; their graphs make a distinct shape called a parabola and quadratic equations are used to model situations in which one variable varies as the square of another. 	What is a quadratic function?	Vocabulary I. Quadratic
 There are many ways to solve a quadratic equation; the method chosen in a specific case depends on the information that is given and the preference of the mathematician. There will be one, two, or no real solutions/roots, which are the x-intercepts of the graph. 	 How can we find solutions to quadratics and how does the graph of a quadratic function relate to its algebraic equation? 	 Binomial Polynomial Quadratic Formula
• Equivalent representations of a function highlight different properties. We can use the tools of algebra to move between different forms. All graphs of quadratic functions are transformations of the parent function: $y=x^2$.	 How can we represent quadratic functions, and why do we use different representations? 	 Complete the square Factor
 Finding zeros of a polynomial allows us to use factoring to separate the components of the equation into simpler pieces. 	 Why is it important to use factoring and other algebraic tools to re- arrange polynomials? 	7. Square root8. Parabola
 Quadratic equations arise from problems involving areas of rectangles. 	 Where does the word quadratic come from, and why do we use it to describe functions involving x²? 	9. Root 10. Parent
 The square roots of negative numbers are pure imaginary numbers and all are multiples of √-1; i² is defined as -1. If there are no real solutions to a quadratic, there are solutions, using i on the set of complex numbers. 	 What can we do when we have no real solutions to a quadratic; what is i? 	Function 11. Reflection 12. Line of Symmetry
 Critical Content The graph of y = ax² + bx + c, when a does not equal 0, is a parabola that opens upward if a > 0 and downward if a < 0. The Quadratic Formula gives the solutions to any quadratic equation in standard form whose coefficients are known. Factoring and completing the square also are ways to solve quadratics You can determine whether the solutions to a quadratic equation with real coefficients are real or not real by calculating a value called the discriminant of the quadratic End behavior of polynomials 	 Key Skills Solve quadratics through factoring, completing the square, and the quadratic formula Graph quadratics, and write equations given a quadratic graph Perfect squares are useful in solving quadratics because they allow us to factor the polynomial. Describe end behavior of polynomials Create a quadratic math model and solve problems given a situation 	13. Vertex

Unit 6: Logarithmic Functions

 Essential Understandings Log functions are inverses of exponential functions; an inverse function is a function that "undoes" another function; if f(x) maps x to y, then f-1(x) maps y back to x. 	 Compelling Questions How do logarithms relate to exponents? 	
 We can rewrite the laws of exponents to work with logarithms and solve logarithmic equations. 	How can we solve logarithmic equations?	Vocabulary
 The graph of a log functions has a characteristic shape and behavior, which is an excellent model for certain situations. 	 What do the graphs of log functions look like and how do they behave? 	 Logarithm Inverse Translation Change of Base Common log Natural log Asymptote
Critical Content	Key Skills	
Laws of logs	 Manipulate log expressions by using the laws of logs 	
Change of base formula	Solve log equations	
Graphs of logs	Graph log functions and write equations given a log graph	
	Create a logarithmic math model and solve problems given a situation	

Unit 7: Statistics

 Essential Understandings The way that data is collected, organized and displayed influences interpretation. Visual models illustrate the correlation of bivariate data. Different kinds of visual models and graphs can reveal different aspects of a situation; we can choose which model to use depending on what we are trying to communicate, what we are trying to predict, or what guestion we are trying to answer. 	 Compelling Questions How can you collect, organize, and display data? How do you interpret the data you have collected? How do charts, tables, and graphs help you interpret data? What kinds of questions can be answered using different data displays, and what data display is appropriate for a given set of data? 	 Vocabulary I. Distribution 2. Variance 3. Measure of central tendency 4. Bivariate 5. Correlation
 The probability of an event's occurrence can be predicted with varying degrees of confidence. The accuracy of a prediction increases with the number of events considered; probability calculations can be applied to solve problems and make decisions Measures of center are used to interpret univariate data. Mean, median, mode, and range can be used to describe the shape of data; the shape gives us insight into the story told by the data. 	 How is the likelihood of an event determined and communicated? How can we use or test our predictions? Are they valid? Are they significant? What mathematical tools can we use to describe data? 	 6. Causation 7. Standard Deviation 8. Quartile 9. Mean 10. Median 11. Mode
 We can use statistical data to support an argument about real and abstract situations, and we can compare our model to actual experimental results. 	 How do we use evidence to support arguments? How do we interpret evidence in order to support arguments? How do we create, test and validate a model? 	12. Range
Critical Contant	Kov Skille	
 Measures of central tendency Analysis of visual models of data Normal (and binomial?) distribution Correlation/ causation 	 New Skills Interpret data using measures of center and spread; calculate and interpret mean, median, mode, range Create, analyze, and interpret visual models of data and graphs. Use technology to find regression functions Model data using familiar functions, and make connections between probability and statistics 	

Unit X: Problem-Solving

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 Essential Understandings A problem solver can develop specific strategies for how to understand and begin a task. There can be different strategies to solve a problem, but some are more effective and efficient than others are. The context of a problem determines the reasonableness of a solution. The ability to solve problems is the heart of mathematics. A problem solver understands what has been done, knows why the process was appropriate, and can support it with reasons and evidence. Working independently is essential to develop personal problem solving skills; working collaboratively can yield deeper and more complex perspectives than working alone. Each is important and related skills can be cultivated and practiced Precision in language and computation is essential to arriving at clear and reliable answers 	 Compelling Questions How do I know where to begin when solving a problem and what do I do when I get stuck?? How do I decide what strategy will work best in a given problem situation? How do I know when a result is reasonable? What is mathematics really about? How does explaining my process help me to understand a problem's solution better? When should I work independently, and when should I collaborate? What is the relationship between solving problems and computation? 	 Vocabulary 1. Perseverance 2. Growth/Fixed Mindset 3. Process 4. Solution 5. Strategy 6. Polya's 4 step process
Critical Content • The content for this unit lies in understanding how to approach novel problems and situations, how and when to use resources, when to be independent. and when to collaborate.	 Key Skills Metacognition Perseverance Independent work when appropriate Collaboration when appropriate Develop strategies for what to do when presented with novel situations Cultivate mathematical habits of mind 	